CANT 2009

New York Number Theory Seminar
Seventh Annual Workshop on
Combinatorial and Additive Number Theory

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Abstracts of lectures

Kent Boklan, Queens College (CUNY)
"Every multiple of 4 except 212, 364, 420, and 428 is the sum of seven cubes"
Abstract: It is conjectured that every integer $N > 454$ is the sum of seven nonnegative cubes. We prove the conjecture when N is a multiple of 4. This is joint work with N.D. Elkies.

Mei-Chu Chang, University of California-Riverside
"Some consequences of the Polynomial Freiman-Ruzsa Conjecture"
Abstract: Assuming the weak polynomial Freiman-Ruzsa conjecture, we derive some consequences on the sum-product problem and the growth of subsets of $SL_3(\mathbf{C})$.

Mohamed El Bachraoui, United Arab Emirates University
"Partitions with relativey prime parts"
Abstract: We discuss some results on integer partitions whose parts are relatively prime. We give an exact formula for the number of relatively prime partitions having exactly three parts, we deduce the parity of this number and as a consequence we determine the parity of the number of patitions into three parts. If time permits, we present the generating function for relatively prime partitions along with some applications.

Simon Griffiths, University of Montreal
"Subset sums in finite abelian groups"
Abstract: For a subset A of a finite abelian group G we write $\Sigma(A) = \{\sum_{b\in B} b : B \subset A\}$ for the set of all subset sums of A. In general $\Sigma(A)$ may be no larger than A, e.g. if A is a subgroup. We note that in this case $\Sigma(A)$ has a large stabiliser, namely itself. What happens in the other extreme case when the stabiliser of $\Sigma(A)$ is trivial? In this case it was proved by DeVos, Goddyn, Mohar and Šámal that $|\Sigma(A)|$ is at least quadratic in $|A|$; specifically they proved that $|\Sigma(A)| \geq |A|^2/64$. After introducing this result we shall discuss a recent proof that gives the asymptotically best possible bound $|\Sigma(A)| \geq (\frac{1}{4} - o(1))|A|^2$.

Li Guo, Rutgers University - Newark
"Euler's formulas for double zeta values and double shuffle relations of multiple zeta values"
Abstract: Multiple zeta values (MZVs) are multi-variable generalizations of the special values of the Riemann zeta function at integers. Their study in the two variable case could be traced back to Euler, but the general study was started by Hoffman and Zagier in early 1990s. A basic question here is to determine the relations among the MZVs. We consider two formulas of Euler, his Decomposition Formula and Sum Formula, and discuss their multi-variable generalizations.

Mariah Hamel, University of Georgia
"Sums of sets of primes with positive relative density"
Abstract: We will show that if A is a subset of the primes with positive relative density, then $A+A$ must have comparable positive density in $\mathbb{Z}$. Our proof combines Fourier analytic techniques of Green and Green-Tao with a combinatorial result on sumsets of subsets of the multiplicative group of integers modulo m. This is joint work with Karsten Chipeniuk.

Peter Hegarty, Chalmers University of Technology and University of Gothenburg, Sweden

"A generalised Erdős-Cameron conjecture"

Abstract : The so-called Erdős-Cameron conjecture is one of the best known problems in the theory of sumsets. In its simplest form, it asserts that the number of sum-free subsets of $\{1, ..., n\}$ is $O(2^{n/2})$. This result has in recent years been proven independently by Green and Sapozhenko, building on several earlier attempts. It is less well-known that the natural generalisation of the conjecture to so-called (k, l)-sum free sets has been proven (finally by Lev) for all but finitely many pairs (k, l), using somewhat different techniques. Also not so well known is the original paper of Erdős and Cameron, in which they were interested in a more general problem. In this talk I wish to propose a natural generalisation of the Erdős-Cameron conjecture, in both a "weak" and a "strong" form, to sets of integers avoiding solutions to an arbitrary non-invariant linear equation. I will show that the (independently interesting) graph theoretical methods which have been a key tool in the attacks on the classical problem can also be used to prove the weak form of the conjecture for some other previously studied equations.

Charles Helou, Pennsylvania State University - Brandywine

"Asymptotic rate and the number of representations"

Abstract: For a strictly increasing sequence A of natural numbers, the asymptotic number of representations is the upper limit of the average number of representations as sum of two elements of A. The caliber of A with respect to another sequence B is the lower limit of the ratio of the n-th term of A to that of B. The basic properties of these notions and the relations between them provide a link between the order of the number of representations and the comparative size of the terms, allowing to extend some classical results.

Le Thai Hoang, UCLA

"Green-Tao theorem in function fields"

Abstract: We adapt the proof of the Green-Tao theorem on arithmetic progressions in primes to the setting of polynomials over a finite fields $\mathbf{F}_q$ to show that, for every k, the irreducible polynomials in $\mathbf{F}_q[t]$ contain configurations of the form $\{f + Pg : (P) < k\}, g \neq 0$. Consequently, the monic irreducible polynomials in $\mathbf{F}_q[t]$ contain affine spaces of arbitrarily high dimension.

Alex Iosevich, University of Missouri - Columbia

"Point configurations in discrete, continuous and arithmetic settings"

Abstract: We are going to discuss various aspects of the classical problem of showing that a sufficiently "large" subset of a given vector space contains a substantial portion of all finite point configurations up to congruence.

Renling Jin, College of Charleston
"Freiman's 3k-3+b conjecture for almost bi-arithmetic progressions"
Abstract: A finite set $B = I_1 \cup I_2 \subseteq \mathbb{Z}$ is called a bi-arithmetic progression of length l if $|B| = l$, I_1 and I_2 are two non-empty arithmetic progressions with the same difference, and I_1+I_1, I_1+I_2, and I_2+I_2 are pairwise disjoint. Freiman conjectured that for a sufficiently large finite set A of k integers, if $|A+A| = 3k-3+b$ for $0 \leq b < \frac{1}{3}k-2$, then A must be a subset of either an arithmetic progression of length at most $2k-1+2b$ or a bi-arithmetic progression of length at most $k+b$. Freiman proved that if A is a sufficiently large non-trivial subset of a bi-arithmetic progression, then A satisfies the $3k-3+b$ conjecture. We generalize this result to the following theorem: There is a positive real ϵ such that if A is a sufficiently large non-trivial subset of an ϵ–almost bi-arithmetic progression, then A satisfies the $3k-3+b$ conjecture. A set $B = I_0 \cup I_1 \cup I_2 \subseteq \mathbb{Z}$ is called an ϵ–almost bi-arithmetic progression if I_0, I_1, and I_2 are disjoint arithmetic progressions of the same difference, $I_1 \cup I_2$ is a bi-arithmetic progression, and $|I_0|/|B| < \epsilon$. This result is one of the steps towards the complete solution of Freiman's $3k-3+b$ conjecture.

Urban Larsson, Chalmers University of Technology and University of Gothenburg, Sweden
"Pairs of m-complementary Beatty Sequences and Wythoff's game"
Abstract: For positive integers k and m, we study a pair of so called m-complementary Beatty sequences:

$$\left(\left\lfloor \frac{n\Phi_{km}}{m} \right\rfloor, \left\lfloor \frac{n(\Phi_{km}+km)}{m} \right\rfloor \right)$$

where

$$\Phi_x = \frac{2-x+\sqrt{x^2+4}}{2},$$

and where n ranges over the non-negative integers. Our main result is that these sequences give the solution to three new extensions of Wythoff's game—of which one has a certain blocking manoeuvre on the rook-type options.

Jaewoo Lee, Borough of Manhattan Community College (CUNY)
"Construction of dense bases of integers with a prescribed representation function"
Abstract: Nathanson showed that almost any function can be the representation function of an asymptotic basis for the integers, and asked how dense such a basis can be. Given a representation function, I will construct asymptotic bases of integers which are maximally dense infinitely often.

Neil Lyall, University of Georgia
"Simultaneous optimal polynomial return times"
Abstract: I plan to discuss some joint work with Akos Magyar concerning certain arithmetic properties of dense sets of integer points.

Steven J. Miller, Williams College
"When almost all sets are difference dominated"
Abstract: Given a finite set of integers A, we can look at the size of its sumset $A+A = \{a_1 + a_2 : a_i \in A\}$ and its difference set $A - A = \{a_1 - a_2 : A_i \in A\}$. As a general pair gives one sum but two differences (since addition is commutative), we expect $|A - A|$ to be larger than $|A + A|$. It was therefore a surprise when Martin and O'Bryant proved a positive percentage of subsets of $\{1, ..., N\}$ have $|A + A|$ larger as $N \to \infty$. In this talk we'll give some new constructions of infinite families where $|A + A| > |A - A|$, and show that our intuition can be salvaged if we change our model of how we look at subsets. We'll see that, in these models, $|A - A|$ almost surely is twice as large as $|A + A|$, and we'll see some fascinating phase transition behavior. This work is joint with Peter Hegarty, Brooke Orosz and Dan Scheinerman.

Rishi Nath, York College (CUNY)
"Some combinatorial properties of 3-cores"
Abstract: A 3-core partition is one in which no hook of length 3 appears. This talk will present some new combinatorial properties of 3-cores obtained by studying the relationship between their maximal arm and leg lengths on the abacus.

Melvyn B. Nathanson, Lehman College (CUNY)
"Geometric group theory and additive number theory"
Abstract: This will be a survey of problems and results arising from the application of ideas in geometric group theory to combinatorial and additive number theory.

Hoi H. Nguyen, Rutgers University-New Brunswick
"On squares in sumsets"
Abstract: A finite set A of integers is square-sum-free if there is no subset of A sums up to a square. We show that the largest cardinality of a square-sum-free subset of $[n]$ is $O(n^{1/3+o(1)})$. (Joint work with V. Vu.)

Lan Nguyen, University of Michigan
"Quantum integers and the solutions of some functional equations arising from their multiplication"
Abstract: In this talk, I will discuss some basic facts concerning quantum integers and show, under some condition on the support base, that the solutions of these functional equations are generated by quantum integers when the fields of coefficients are of characteristic zero. Also, I would show some cases where no such generation is possible.

Kevin O'Bryant, College of Staten Island (CUNY)
"Sets of integers without long arithmetic progressions"
Abstract: Behrend's 1946 construction of subsets of $\{1, 2, ..., n\}$ that do not contain 3 elements in arithmetic progression, and Rankin's 1960 generalization to longer APs, were the densest known until 2008. In 2008, Elkin strengthened Behrend's construction, and then Green and Wolf simplified Elkin's work. This talk will present the ideas behind this recent progress and also the extension to longer APs.

Brooke Orosz, Essex County College
"h-fold sum and difference Sets"
Abstract: The study of sum sets is a central part of additive number theory. I will look at sets such as the 3-fold sum set, and compare the size of $A + A + A$ to $A+A-A$. In particular, if f, g are linear forms with h terms, and all coefficients are ± 1, do there always exist sets A, B, C such that $|f(A)| > |g(A)|$, $|f(B)| < |g(B)|$ and $|f(C)| = |g(C)|$? I suspect the answer is yes in general, but I can only prove it in certain cases.

Jonathan Sondow, New York
"Ramanujan Primes and Bertrand's Postulate"
Abstract: The nth Ramanujan prime is the smallest natural number R_n such that if $x \geq R_n$, then there are at least n primes in the interval $(x/2, x]$. Bertrand's postulate is $R_1 = 2$. Ramanujan proved that R_n exists and gave the first five values as 2, 11, 17, 29, 41. In this talk, I prove that $2n \log 2n < R_n < 4n \log 4n$ for all n, and that R_n is asymptotic to the $2n$th prime. I also estimate the length of the longest string of consecutive Ramanujan primes among the first n primes, explain why there exist more twin Ramanujan primes than expected, and make three conjectures. My paper is to appear in the August 2009 issue of the Monthly.

Craig Spencer, Institute for Advanced Study
"Roth's theorem in function fields"
Abstract: In this talk, we will discuss how the circle method can be used to prove variants of Roth's theorem in function fields.

Mario Szegedy, Rutgers University-New Brunswick
“ Algorithms to tile the infinite grid with finite clusters”
Abstract: We say that a (finite) subset T of the two dimensional infinite grid tiles the grid, if we can decompose the latter to non-overlapping translates of T. No algorithm is known to decide whether a finite T is a tiler in the above sense or not. Here we present two algorithms, one for the case when $|T|$ is prime, and another for the case when $|T| = 4$. Both algorithms generalize to the case, where we replace the grid with an arbitrary finitely generated abelian group. Our result partially settles the Periodic Tiling Conjecture raised by J. Lagarias and Y. Wang that every tiler T of the grid has a periodic co-tiler, and we also get the following generalization of a theorem of Redei:

Theorem 1. *Let G be a (finite or infinite) abelian group G with a generator set T of prime cardinality such, that 0 is in T, and there is a subset T' of G with the property that for every $g \in G$ there are unique $t \in T$, $t' \in T'$ such that $g = t + t'$. Then T' can be replaced with a subgroup of G that also has the above property.*

Benjamin Weiss, University of Michigan
“Galois groups of p-adic polynomials of fixed degree”
Abstract: The space of fixed degree polynomials with p-adic coefficients has a natural probability distribution. Each polynomial also has an associated group which is the Galois group of its splitting field. We will discuss the induced distribution on groups, and derive results for the limiting distribution as p grows. Time permitting, we will discuss a relationship to Serre’s mass formulae for extensions of local fields, and prove a complementary theorem to the Chebotarev Density theorem. This work is joint with Chris Hall.

Julia Wolf, Rutgers University-New Brunswick
“The minimum number of monochromatic 4-term progressions”
Abstract: We derive bounds on the minimum number of monochromatic 4-term arithmetic progressions in any 2-colouring of $\mathbf{Z}/p\mathbf{Z}$. In the process we touch upon the subject of quadratic Fourier analysis, as well as a related question in graph theory.

Gang Yu, Kent State University
“A graph labeling and some additive problems”
Abstract: A tree with n vertices is called a Leech tree if one can assign to each edge a weight so that the weighted lengths of the $n(n-1)/2$ paths are exactly $1, 2, \ldots, n(n-1)/2$. It is conjectured that there are only finitely many Leech trees. In this talk, I will introduce some partial results (joint with Jian Shen). I will also talk about some related problems in additive number theory, including a new upper bound for the maximum size of finite $B_2[g]$ sets.