CANT 2010

New York Number Theory Seminar
Eighth Annual Workshop on Combinatorial and Additive Number Theory

CUNY Graduate Center
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Abstracts of lectures

Hannah Alpert, University of Chicago
"Finite phase transitions in countable abelian groups"
Abstract: Let A be an infinite set that generates a group G. The sphere $S_A(r)$ is the set of elements of G for which the word length with respect to A is exactly r. We say G admits all finite transitions if for every $r \geq 2$ and every finite symmetric subset $W \subset G \setminus \{e\}$, there exists an A with $S_A(r) = W$. We determine which countable abelian groups admit all finite transitions, and also show that $\mathbb{R}^n$ and the finitary symmetric group on $\mathbb{N}$ admit all finite transitions.

Paul Baginski, Université Claude Bernard Lyon, France
"Lengths of factorizations in numerical semigroup rings"
Abstract: Given a ring, $(R, +, \cdot)$, and a semigroup, $(S, +)$, one can construct a new ring, $R[S]$, called the semigroup ring. This ring generalizes the standard polynomial ring $R[X]$. Semigroup rings have been studied extensively, especially in the case where $R = \mathbb{Z}$ or a field. The semigroup ring, in an intuitive sense, carries some of the factorization structure of S, but due to the interaction with R, one often gets new behavior for factorization. We will discuss to what extent the factorization properties of R and S can be separated within the semigroup ring when S is a numerical monoid. In particular, we will consider how longest factorizations of elements can be estimated in terms of factorizations within R and S separately.

Gautami Bhowmik, Université Lille, France
"Zero-sum problems of elementary p-groups"
Abstract: For a sufficiently large prime number p, a positive integer d and a subset A of $\mathbb{Z}_p^d$, we study the minimal size of A such that it contains a sequence of elements that add up to zero. Inversely, we study the properties of maximal zero-sum free sets. The results illustrate the use of classical and new methods of combinatorics, exponential sums and geometry of numbers.

Kent Boklan, Queens College (CUNY)
"Every even except 22, 50, 114, 186, 212, 238, 364, 420, 428 and 454 and every odd congruent to 0, 1, 2, 7, 8 (mod 9) is the sum of seven cubes"
Abstract: It is conjectured that every integer $N > 454$ is the sum of seven non-negative cubes. At CANT 2009 we established that every multiple of 4 except 212, 364, 420, and 428 is the sum of seven cubes. This year we prove the conjecture when N is even and N is odd and congruent to 0,1,2, 7 and 8 (mod 9). (Joint work with N. D. Elkies.)

Mei-Chu Chang, University of California-Riverside
"Incomplete character sums over finite fields"
Abstract: We present various new estimates on incomplete character sums over finite fields. In particular, we consider polynomials in several variables that factor completely over an extension field and certain mixed character sums. Several of these issues go back to Burgess' work.

Scott Chapman, Sam Houston State University
"Erdős-Zaks all divisor sets"
Abstract: Let $\mathbf{Z}_n$ be the finite cyclic group of order n and $S \subseteq \mathbf{Z}_n$. We examine the factorization properties of the Block Monoid $\mathcal{B}(\mathbb{Z}_n, S)$ when S is constructed using a method inspired by a 1990 paper of Erdős and Zaks. For such a set S, we develop an algorithm in Section 2 to produce and order a set $\{\mathfrak{M}_i\}_{i=1}^{n-1}$ which contains all the non-primary irreducible Blocks (or atoms) of $\mathcal{B}(\mathbb{Z}_n, S)$. This construction yields a weakly half-factorial Block Monoid. After developing some basic properties of the set $\{\mathfrak{M}_i\}_{i=1}^{n-1}$, we examine in Section 3 the connection between these irreducible blocks and the Erdős-Zaks notion of "splittable sets." In particular, the Erdős-Zaks notion of "irreducible" does not match the classic notion of "irreducible" for the commutative cancellative monoids $\mathcal{B}(\mathbb{Z}_n, S)$. We close in Sections 4 and 5 with a detailed discussion of the special properties of the blocks $\mathfrak{M}_1$ with an emphasis on the case where the exponents of $\mathfrak{M}_1$ take on extreme values. The work of Section 5 allows us to offer alternate arguments for two of the main results of the original paper by Erdős and Zaks.

Brian Cook, University of British Columbia
"Constellations in P^d"
Abstract: A constellation is a higher dimensional analogue of an arithmetic progression, namely something of the shape $\{\mathbf{x}, \mathbf{x}+t\mathbf{e_1}, ..., \mathbf{x}+t\mathbf{e_d}\} \in \mathbf{Z}^d$, where $t \in \mathbf{Z}$ and $\mathbf{x}, \mathbf{e_1}, ..., \mathbf{e_d} \in \mathbf{Z}^d$. We discuss finding such patterns lying inside a relatively dense subset of P^d, where P denotes the set of primes. While the case for general sets of $\{\mathbf{e_j}\}$ remains open, if the i^{th} coordinate of the $\mathbf{e_j}$ is distinct in j for each i, the existence of infinitely many constellations of this shape is shown. This work is joint with A. Magyar.

David Covert, University of Missouri
"A Furstenberg-Katznelson-Weiss type theorem on $(d+1)$-point configurations in finite fields"
Abstract: We show that if $E \subset \mathbb{F}_q^d$, the d-dimensional vector space over the finite field with q elements, and $|E| \geq \rho q^d$, where $q^{-\frac{1}{2}} \ll \rho \leq 1$, then E determines an isometric copy of at least $c\rho^{d-1}q^{\binom{d+1}{2}}$ distinct $(d+1)$-point configurations.

Aviezri S. Fraenkel, Weizmann Institute of Science, Israel
"Complementary sets of integers"
Abstract: Old and new results, problems and conjectures on sets that split the integers. From Leopold Kronecker (1884), Lord John William Strutt Rayleigh (1894), Samuel Beatty (1926), Alexander Markowitsch Ostrowski (1927), James V. Uspensky (1927), up to Ron Graham (1963), (1973), Thoralf Albert Skolem (1957), Thøger Bang (1957), Ryozo Morikawa (1985), Robert Jamie Simpson (1986, 1991, 2004), Robert Tijdeman (2000), Kevin O'Bryant (2002), Graham and O'Bryant (2005) and many others. Emphasis is on contemporary results such as fractal-like and fractional complementary sets of integers, including a 37-year old conjecture that has been solved for the integers and for the irrationals, yet is wide open for the rationals. Time permitting, applications to combinatorial games will be indicated. In part, joint work with Peter Hegarty and Urban Larsson.

John Friedlander, University of Toronto
"Close relatives of the primes"

John Griesmer, University of British Columbia
"Sumsets with one dense summand"
Abstract: Renling Jin proved that $A+B$ is piecewise syndetic whenever A and B are sets of integers with positive upper Banach density. This result was generalized and strengthened by Jin, Keisler, Bergelson, Furstenberg, Weiss, Fish, and Beiglbock, in various combinations. We show that one may weaken the hypothesis on A; a special case of our result says that if A has positive relative density in the set $\{[n^5/2] : ninN\}$, where [] is the floor function, then $A + B$ is piecewise syndetic whenever B has positive upper Banach density. We give some contrasting examples and show that one can make nontrivial conclusions assuming only that A is infinite and B has positive upper Banach density. Our main tool is ergodic theory.

Sinan Gunturk, Courant Institute, NYU
"Additive number theory and quantization of signals"
Abstract: Sums of the form $\sum \pm v_k$, where $v_1, \ldots, v_N$ are a given collection of N vectors in $\mathbb{R}^d$, provide a model for quantized linear representations of finite dimensional signals. I will present various results on the local covering radii of such sums in certain special settings of interest. This is ongoing joint work with V. Molino.

Peter Hegarty, Chalmers University of Technology and University of Gothenburg, Sweden
"Some new connections between additive number theory and graph theory"
Abstract : One of the well-known points of intersection between additive number theory and graph theory is the use of Cayley sum graphs. In this talk, I will describe two recent works in which we exploit this connection to pose some new problems and prove some new results which may be of interest both to number theorists and graph theorists. The first work starts with a (not so well known) problem in graph theory, concerned with so-called exponent sets of digraphs, and specialises it to the case of Cayley sum graphs, where it becomes a problem about the possible orders of a basis for a finite (cyclic) group. Our basic result is that the set of possible basis orders exhibits large gaps. The second work starts, conversely, with a (very well-known) result in additive number theory, the Cauchy-Davenport theorem, considers its formulation in terms of Cayley sum graphs and then seeks to prove a similar result for arbitrary regular graphs. Here we prove what we believe is a fundamental result about regular graphs, which seems to have escaped attention up to now. This result is only a first step, however, and many open problems remain. (The first part of the talk is joint work with Peter Dukes and Sarada Herke).

Charles Helou, Pennsylvania State University - Brandywine
"Supremum of representation functions"
Abstract: For a subset A of $\mathbf{N} = \{0, 1, 2, \ldots\}$, the representation function of A is defined by $r_A(n) = |\{(a, a') \in A \times A : a + a' = n\}|$ for $n \in \mathbf{N}$, where $|E|$ denotes the cardinality of a set E. Its supremum is the element $s(A) = \sup\{r_A(n) : n \in \mathbf{N}\}$ of $\overline{\mathbf{N}} = \mathbf{N} \cup \{\infty\}$. Interested in the question "when is $s(A) = \infty$? ", we study some properties of the function $A \mapsto s(A)$, determine its range, and construct some subsets A of $\mathbf{N}$ for which $s(A)$ satisfies certain prescribed conditions. (Joint work with Grekos, Haddad, Pihko).

Alex Iosevich, University of Rochester
"Sharpness of Falconer's estimate, geometric incidence theorems, and distribution of lattice points in convex domains, I"
Abstract. In the paper introducing the celebrated Falconer distance problem, Falconer proved that the Lebesgue measure of the distance set is positive, provided that the Hausdorff dimension of the underlying set is greater than $(d+1)/2$. His

result is based on the estimate

(1) $$\mu \times \mu(x, y) : 1 \leq |x - y| \leq 1 + \varepsilon$$

where μ is a Borel measure satisfying the energy estimate

$$I_s(\mu) = \int \int Is(?) = |x - y|^{-s} d\mu(x) d\mu(y) < \infty$$

for $s > (d+1)/2$. An example due to Mattila shows in two dimensions that for no $s < 3/2$ does $I_s(\mu) < 1$ imply (1). His construction can be extended to three dimensions. Mattila's example readily applies to the case when the Euclidean norm in (1) is replaced by a norm generated by a convex body with a smooth boundary and non-vanishing Gaussian curvature. We prove, for all $d \geq 2$, that for no $s < (d+1)/2$ does $I_s(\mu) < 1$ imply (1) or the analogous estimate where the Euclidean norm is replaced by the norm generated by a particular convex body B with a smooth boundary and everywhere non-vanishing curvature. Our construction, based on a combinatorial construction due to Pavel Valtr, naturally leads us to some interesting connections between the problem under consideration, geometric incidence theorem in the discrete setting and distribution of lattice points in convex domains. We also prove that Mattilas example can be discretized to produce a set of points and annuli for which the number of incidences is much greater than in the case of the lattice. In particular, we use the known results on the Gauss Circle Problem and a discretized version of Mattila's example to produce a non-lattice set of points and annuli where the number of incidences is much greater than in the case of the standard lattice.

Renling Jin, College of Charleston

"Every compact plane set with the square property contains an integral diagonal pair"

Abstract: A pair of plane vectors (x, y) is called an integral pair if both vertical and horizontal coordinates of $x - y$ are integers. An integral pair (x, y) is called diagonal if both coordinates of $x - y$ are non-zero. A compact plane set K is said to have the square property if for any $x \in \mathbf{R}^2$ there exists $y \in K$ such that (x, y) is an integral pair. Note that the unit square (with boundary) is a compact plane set with the square property. In the last problem session of CANT2009 M. Nathanson mentioned the following question originally raised by P. Hegarty: "Can we find a plane compact set with the square property, which contains no integral diagonal pairs?" We will present a negative answer to the problem and sketch a proof.

William J. Keith, Drexel University

"Partitions with prescribed hooksets"

Abstract: A t-core is a partition with hook lengths required to be nonmultiples of t. Here we explore other prescriptions on the hookset of a partition. What are necessary and sufficient conditions for a partition to have only hooks from an arbitrary set S? When will a partition have many hooks from such a set? Which arbitrary finite multisets of natural numbers can be the multisets of hooks of a partition, and how many of these are there compared to the number of partitions?

Alex Kontorovich, IAS and Brown
"Progress on affine sieves"
Abstract: We will discuss recent progress with Jean Bourgain on the Affine Sieve, which aims to find primes or almost-primes in sets of integers generated by group actions, with applications to prime entries in matrix groups.

Brandt Kronholm, SUNY at Albany
"Generalized Ramanujan congruence properties of the restricted partition function $p(n,m)$"
Abstract: The restricted partition function $p(n,m)$ enumerates the number of partitions of n into exactly m parts. The relationship between the unrestricted partition function $p(n)$ and $p(n,m)$ is clear: $p(n) = p(n,1) + p(n,2) + ... + p(n,n)$. In 1919 Ramanujan observed proved the following partition congruences:

$$p(5n+4) \equiv 0 \pmod 5$$
$$p(7n+5) \equiv 0 \pmod 7$$
$$p(11n+6) \equiv 0 \pmod{11}$$

Ono (2000) proved that there are such congruences for $p(n)$ modulo every prime $\ell > 3$. Ramanujan further conjectured a generalization for a modulus of powers of 5, 7, and 11. In this talk we will discuss a Ramanujan-like congruence relation for $p(n,m)$ where for our choice of any prime power modulus ℓ^α, there is no restriction on n.

Urban Larsson, Chalmers University of Technology and University of Gothenburg, Sweden
"Invariant and Dual subtraction games resolving the Duchêne-Rigo Conjecture"
Abstract: We prove a recent conjecture of Duchêne and Rigo, stating that every complementary pair of homogeneous Beatty sequences represents the solution to an *invariant* impartial game. Here invariance means that each available move in a game can be played anywhere inside the game-board. In fact, we establish such a result for a wider class of pairs of complementary sequences, and in the process generalize the notion of a *subtraction game*. Given a pair of complementary sequences (a_n) and (b_n) of positive integers, we define a game G by setting $\{\{a_n, b_n\}\}$ as invariant moves. We then introduce the invariant game $G^\star$, whose moves are all non-zero P-positions of G. Provided the set of non-zero P-positions of $G^\star$ equals $\{\{a_n, b_n\}\}$, this *is* the desired invariant game. We give sufficient conditions on the initial pair of sequences for this 'duality' to hold. This is joint work with Peter Hegarty and Aviezri Fraenkel.

Jaewoo Lee, Borough of Manhattan Community College (CUNY)
"Algebraic proof for the geometric structure of sumsets"
Abstract: A few years ago, we presented a proof of how sumsets grow geometrically inside convex hulls of underlying sets. That proof was geometric in nature. In this talk, we will present an algebraic proof for their geometric structures.

Željka Ljujić, CUNY Graduate Center
"Periodicity of complementing multisets"
Abstract: Let A be a finite multiset of integers. If B be a multiset such that A and B are t-complementing multisets of integers, then B is periodic. We obtain the Biro-type upper bound for the smallest such period of B: Let $\varepsilon > 0$. We assume that $\operatorname{diam}(A) \geq n_0(\varepsilon)$ and that $\sum_{a \in A} w_A(a) \leq (\operatorname{diam}(A) + 1)^c$, where c is any constant such that $c < 100 \log 2 - 2$. Then B is periodic with period $\log k \leq (\operatorname{diam}(A) + 1)^{\frac{1}{3}+\varepsilon}$.

Neil Lyall, University of Georgia
"Simultaneous optimal polynomial recurrence"
Abstract: We will discuss some specific (single) recurrence properties of measure preserving probability systems and (quantitative versions of) their combinatorial consequences.

Steven J. Miller, Williams College
"Cookie Monster meets the Fibonacci numbers. Mmmmmm – theorems!"
Abstract: A beautiful theorem of Zeckendorf states that every positive integer can be written uniquely as a sum of non-consecutive Fibonacci numbers. Once this has been shown, it is natural to ask how many Fibonacci numbers are needed. Lekkerkerker proved that the average number of such summands needed for integers in $[F_n, F_{n+1})$ is $n/(\gamma^2+1)$, where γ is the golden mean. We present a combinatorial proof of this through the cookie problem and differentiating identities, and show how this technique will yield an Erdos-Kac type result that the number of summands is Gaussianly distributed. These and related problems will be investigated by the SMALL 2010 REU students at Williams College.

Rishi Nath, York College (CUNY)
"Welter's game and Young diagrams: A new interpretation involving hooks"
Abstract: Welter's game is a two-player combinatorial game that has been completely analyzed by J. Conway and others in its original formulation. A two-dimensional interpretation of this game involving Young diagrams and alternating hook removals is presented. While some unexpected results have been obtained via this perspective, it remains to understand how the known theory translates into the context of hook removals.

Melvyn B. Nathanson, Lehman College (CUNY)
"Indecomposable systems and a theorem of de Bruijn in additive number theory"
Abstract: This paper gives a complete proof of a theorem of de Bruijn that classifies finite or infinite systems $\mathcal{A} = \{A_i\}_{i \in I}$ of sets of nonnegative integers such that $0 \in A_i$ for all i and every nonnegative integer can be written uniquely in the form $\sum_{i \in I} a_i$ with $a_i \in A_i$ for all i and $a_i \neq 0$ for only finitely many i. All indecomposable unique representation systems are determined.

Lan Nguyen, University of Michigan
"Functional equations for quantum integers"
Abstract: Quantum integers are used in the study of the q-analogues of classical zeta functions, the study of representations of quantum groups, and in many other contexts. The arithmetic of these integers is studied by Melvyn Nathanson, Yang Wang, Alex Borisov, and others. In particular, they study the solutions of certain functional equations arising naturally from the arithmetic of these integers. These solutions can be divided into two basic types, namely those with prime semigroup supports and those with supports not necessarily prime semigroups. Certain results concerning the former solution type are known by the works of Nathanson, Wang, and Borisov in the case where the field of coefficients is the field of rational numbers and by our work in the case where the field of coefficients is of characteristic zero. In particular, we resolved all open problems in the zero characteristic setting for this solution type. In this talk, we present some of our new results concerning the latter type of solutions. These are the first known results concerning this type of solutions, resolving in particular several open problems posed by Nathanson concerning this solution type in characteristic zero setting.

Kevin O'Bryant, College of Staten Island (CUNY)
"Sets of integers without solutions to systems of equations"
Abstract: Let M be an arbitrary $m \times n$ integer matrix. How thick can a set A of integers be under the condition that A^n not contain any nontrivial solutions x to $Mx = 0$? This talk will survey the state of the art regarding constructions of such sets, and identify some of the outstanding special cases.

Alex Rice, University of Georgia
"Polynomial difference in the primes"
Abstract: Given a natural number N, how many pairs of primes less than or equal to N differ by a perfect square? In this talk we utilize the Hardy-Littlewood circle method, including classical estimates from Waring's Problem and Vinogradov's Theorem, to resolve and generalize this problem. Namely, we give an asymptotic formula that counts the number of pairs of primes whose differences lie in the image of a given non-constant polynomial with integer coefficients. The formula is meaningful as long as the image of the polynomial is not entirely odd, and the main term is remarkably explicit in the case of monomials. (Joint work with Neil Lyall.)

Steven Senger, University of Missouri
"Sharpness of Falconer's estimate, geometric incidence theorems, and distribution of lattice points in convex domains, II"
Abstract: This is the second part of a talk on joint work with Alex Iosevich. The abstract appears above under Iosevich.

Jonathan Sondow, New York
"Algebraic and transcendental solutions of some exponential equations"
Abstract: We study algebraic and transcendental powers of positive real numbers, including solutions of each of the equations $x^x = y$, $x^y = y^x$, $x^x = y^y$, $x^y = y$, and $x^{x^y} = y$. Applications to values of the iterated exponential functions are given. The main tools used are classical theorems of Hermite-Lindemann and Gelfond-Schneider, together with solutions to exponential Diophantine equations. This is joint work with Diego Marques. Our paper is to appear in *Annales Mathematicae et Informaticae*.

John Steinberger, Institute for Theoretical Computer Science, Tsinghua University, Beijing
"Coset arrays and nonnegative integer linear combinations"
Abstract: We refine a conjecture of Coppersmith and Steinberger on nonnegative integer linear combinations and prove the new conjecture for three classes of coset combinations, including a class for which the original conjecture was still open.