New York Number Theory Seminar
CUNY Graduate Center
Spring, 2024

INFORMATION

The New York Number Theory Seminar meets every Thursday at 2:30 p.m. EDT (New York time). The schmooze session (to which everyone is invited) begins at 2:30 p.m. The lecture begins at 3:00 p.m.

ZOOM LOGIN:

https://lehman-cuny-edu.zoom.us/j/84066184717?pwd=dkZFbVdyQm5KMUJtcUhFcjMxV0J2QT09

Meeting ID: 840 6618 4717 Passcode: 304403

SCHEDULE OF TALKS

Date: Thursday, February 1 at 3:00 p.m. (on Zoom)
Speaker: Mel Nathanson, Lehman College and CUNY Graduate Center
Title: Finitely many implies infinitely many (for polynomials in infinitely many variables)
Abstract: Many mathematical statements have the following form: Let X be an infinite set of equations. If every finite subset of the equations has a common solution, then the infinite set of equations has a common solution. A result of this type will be described for certain infinite sets of polynomial equations in infinitely many variables. This is joint work with David Ross.

Date: Thursday, February 8 at 3:00 p.m. (on Zoom)
Speaker: David Ross, University of Hawaii
Title: Finitely many implies infinitely many, part 3: the nonstandard version
Abstract: In a pair of recent seminars, Mel Nathanson has discussed proofs, using the Tychonoff Theorem, for existence of solutions to infinite sets of equations in infinitely many variables. In at least one case the proof was an adaptation of an argument using nonstandard analysis. In this talk I'll try to explain such nonstandard arguments, hopefully making them intelligible to mathematicians who haven't seen nonstandard methods before.

Date: Thursday, February 15 at 3:00 p.m. (on Zoom)
Speaker: Florian Luca, University of Witwatersrand and Oxford University
Title: Positive integers k such that $3^k + 1 \equiv 0 \pmod{3k+1}$
Abstract: In my talk we will look at positive integers k such that $3^k + 1 \equiv 0 \pmod{3k+1}$. We show that there are infinitely many such. They are all odd and composite and they have a counting function that is much smaller than the primes. This is work in progress.

Date: Thursday, February 22 at 3:00 p.m. (on Zoom)
Speaker: Sayak Sengupta, Binghamton University (SUNY)
Title: Nilpotent and infinitely nilpotent integer sequences
Abstract: We say that an integer sequence $\{r_n\}_{n\geq 0}$ has a generating polynomial $u(x)$ over $\mathbb{Z}$ if for every positive integer n one has $u^{(n)}(r_0) = r_n$. In addition, if such a sequence satisfies the condition that $r_n = 0$ for some positive integer n (respectively, $r_n = 0$ for infinitely many positive integers n), then we say that $\{r_n\}_{n\geq 0}$ is a nilpotent sequence (respectively, $\{r_n\}_{n\geq 0}$ is an infinitely nilpotent sequence). In this talk we will provide (and discuss) some important characteristics of nilpotent and infinitely nilpotent sequences.

Date: Thursday, February 29 at 3:00 p.m. (on Zoom)
Speaker: Senia Sheydvasser, Bates College
Title: Hidden structures in families of Ulam sequences
Abstract: Stanislaw Ulam defined the original Ulam sequence as follows:
Start with 1,2, and then each subsequent term is the next smallest integer
that is the sum of two distinct prior terms in exactly one way.
(The next few terms are 1,2,3,4,6,8,...) There is now a veritable zoo
of "Ulam-like" sequences and sets, most of which share the main trait of the original:
There is clear numerical evidence that there is an underlying structure,
but for the most part we can prove almost nothing. (As a simple example:
Computation of trillions of terms of the Ulam sequence strongly suggests
that it grows linearly. The best known bound is that it can't grow faster than
exponentially fast.) One of the few partial results that we can prove concerns
what has been termed the Rigidity Conjecture. The original proofs surrounding
this were model-theoretic in nature—what we shall show is that there is a completely
constructive proof using a new variation of Ulam sequences, and the hints toward
a broader solution that this offers.

Date: Thursday, March 7 at 3:00 p.m. (on Zoom)
Speaker: James Sellers, University of Minnesota - Duluth
Title Surprising connections between integer partitions statistics:
The crank, minimal excludant, and partition fixed points
Abstract: A *partition* of an integer n is a finite sequence of positive integers $p_1 \geq p_2 \geq \cdots \geq p_k$ such that $n = p_1 + p_2 + \cdots + p_k$. We let $p(n)$ denote the number of partitions of n. For example, $p(4) = 5$ because there are five partitions of the integer $n = 4$:
4, 3+1, 2+2, 2+1+1, 1+1+1+1
In 1919, just one year before his death, Ramanujan discovered and proved some unexpected, and truly amazing, divisibility properties for the function $p(n)$. Since then, several mathematicians have studied $p(n)$ from different perspectives, trying to better understand these divisibility properties, especially from a combinatorial perspective. In the process, numerous "statistics" have been defined on partitions, including the rank and crank of a partition. In this talk, I will discuss this history in more detail, and then I will transition to some relatively new partition statistics, including the *missing excludant* (or *mex*) of a partition. I will discuss unexpected connections between this mex statistic and the crank, and then we will transition to some very recent work of Blecher and Knopfmacher on partition fixed points which, unbeknownst to them, is very closely connected to the crank and mex statistics. We will close by generalizing this concept of partition fixed points and show how this new family of functions naturally connects with generalized versions of the aforementioned partition statistics.
This is joint work with Brian Hopkins, St. Peter's University.

Date: Thursday, March 14 at 3:00 p.m. (on Zoom)
Speaker: David and Gregory Chudnovsky, NYU
Title: The telephone gossip problem: An hommage to Richard Bumby

Date: Thursday, March 28 at 3:00 p.m. (on Zoom)
Speaker: Mel Nathanson, CUNY
Title: Landau's converse to Hölder's inequality, and other inequalities

Date: Thursday, April 4 at 3:00 p.m. (on Zoom)
Speaker: Mel Nathanson, CUNY
Title: Introductory remarks on Hilbert's inequality and the large sieve
Abstract: Sample results in number theory obtained from the large sieve.

Date: Thursday, April 11 at 3:00 p.m. (on Zoom)
Speaker: Kevin O'Bryant, CSI
Title: B_h-sets
Abstract: Fix a positive integer h. A B_h-set is a set of natural numbers that does not contain x_i, y_i with $x_1 + \cdots + x_h = y_1 + \cdots + y_h$, except for the trivial solutions where $x_1, \ldots, x_h$ is a rearrangement of $x_1, \ldots, x_h$. The primary challenge is to make the k-th largest element of a B_h-set as small as possible. This talk will contain the state of the art for this problem, with special attention to how the problem changes as h grows.

Date: Thursday, April 18 at 3:00 p.m. (on Zoom)
Speaker: Leo Schäfer, Universität Göttingen, Germany
Title: Telling apart coarsifications of the integers
Abstract: We introduce an invariant for coarse groups that is able to differentiate some coarsifications of the integers up to isomorphism. In particular, we will see that coarsifications coming from pro-Q topologies (and therefore also the p-adic topologies) are not isomorphic. Partial results for metrics stemming from Cayley graphs are also obtained, but there remain open questions in this regard. This is joint work with Federico Vigolo.

Date: Thursday, April 25 at 3:00 p.m.
NO SEMINAR

Date: Thursday, May 2 at 3:00 p.m. (on Zoom)
Speaker: Alexander Borisov, Binghamton University
Title: Locally integer polynomial functions
Abstract: A locally integer polynomial function on a subset X of $\mathbb{Z}$ is a function $f : X \to \mathbb{Z}$ such that its restriction to every finite subset is given by a polynomial in $\mathbb{Z}[x]$. I hope to convince you that these objects are interesting and deserve further study. The talk will be based on my recent preprint https://arxiv.org/abs/2401.17955 and on further work in progress on a rather mysterious analogy between locally integer polynomial functions on infinite X and complex analytic functions. Several open questions will be proposed, highlighting how little appears to be known about these seemingly elementary objects.

Date: Thursday, May 9 at 3:00 p.m. (on Zoom)
Speaker: Quentin Dubroff, Rutgers University
Title: tba
Abstract:

Date: Thursday, May 16 at 3:00 p.m. (on Zoom)
Speaker: Florian Luca, University of Witwatersrand, South Africa
Title: On transcendence of Sturmian and Arnoux-Rauzy words
Abstract: We consider numbers of the form $\alpha = \sum_{n=0}^{\infty} \frac{u_n}{\beta^n}$, where (u_n) is an infinite word over a finite alphabet and β is a complex number of absolute value greater than one. We present a combinatorial criterion on u, called echoing, that implies that α is transcendental whenever β is algebraic. We show that every Sturmian word is echoing, as is the Tribonacci word, a leading example of an Arnoux-Rauzy word. We give an application of our transcendence results to the theory of dynamical systems, showing that for a contracted rotation on the unit circle with algebraic slope, its limit set is either finite or consists exclusively of transcendental elements other than its endpoints 0 and 1. This confirms a conjecture of Bugeaud, Kim, Laurent, and Nogueira. Joint work with P. Kebis, A. Scoones, J. Ouaknine and J. Worrell.