CANT 2018 Abstracts

Sixteenth Annual Workshop on Combinatorial and Additive Number Theory

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Sukumar Das Adhikari, Harish-Chandra Research Institute, and Ramakrishna Mission Vivekananda Educational and Research Institute, India
Title: Some early Ramsey-type theorems in combinatorial number theory and some generalizations
Abstract: We start with some classical results in Ramsey Theory and report some recent results on some questions related to monochromatic solutions of linear Diophantine equations. We shall also mention few challenging open questions in this area of Ramsey Theory.

Shabnam Akhtari, University of Oregon
Title: Representation of integers by binary forms
Abstract: Let $F(x, y)$ be an irreducible binary form of degree at least 3 and with integer coefficients. By a well-known result of Thue, the equation $F(x, y) = m$ has only finitely many solutions in integers x and y. I will discuss some quantitative results on the number of solutions of such equations. I will also talk about my recent joint work with Manjul Bhargava, where we show that many equations of the shape $F(x, y) = m$ have no solutions.

Paul Baginski, Fairfield University
Title: Nonunique factorization in the ring of integer-valued polynomials
Abstract: The ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$ is the set of polynomials with rational coefficients which produce integer values for integer inputs. Specifically,

$$\mathrm{Int}(\mathbb{Z}) = \{f(x) \in \mathbb{Q}[x] \mid \forall n \in \mathbb{Z}\, f(n) \in \mathbb{Z}\}.$$

$\mathrm{Int}(\mathbb{Z})$ constitutes an interesting example in algebra from many perspectives; for example, it is a natural example of a non-Noetherian ring. It is also a ring with nonunique factorization. Every element has only finitely many factorizations, yet the number of irreducibles involved grows without bound. Frisch recently demonstrated that in $\mathrm{Int}(\mathbb{Z})$, you can find an element $f(x)$ that has any factorization lengths you desire and you can even prescribe the number of factorizations of each length. The polynomials constructed in this way have high degree. We give a graded analysis, determining all the possible elasticities and catenary degrees for a polynomial as a function of the degree of the polynomial. Joint work with Greg Knapp, Jad Salem, and Gabrielle Scullard.

Gautami Bhowmik, University of Lille, France
Title: Asymptotics of Goldbach functions and zeros of L–functions
Abstract: We show that good asymptotics of the classical Goldbach generating function is equivalent to the Riemann Hypothesis. For the case of primes in arithmetic progression, we obtain information on zeros of Dirichlet L functions.

Margaret Bilu, New York University
Title: Motivic Euler products
The Grothendieck group of varieties over a field k is the quotient of the free abelian group of isomorphism classes of varieties over k by the so-called cut-and-paste relations. It moreover has a ring structure coming from the product of varieties over k. Many problems in number theory have a natural, more geometric counterpart involving elements of this ring. I will explain how some power series with coefficients in the Grothendieck ring can be endowed with an Euler product decomposition analogous to classical Euler products in number theory, and discuss geometric applications of this notion.

Scott Chapman, Sam Houston State University
Title: On the catenary degree of elements in a numerical monoid generated by an arithmetic sequence
Abstract: We compute the catenary degree of elements contained in numerical monoids generated by arithmetic sequences. We find that this can be done by describing each element in terms of the cardinality of its length set and of its set of factorizations. As a corollary, we find for such monoids that the catenary degree becomes fixed on large elements. This allows us to define and compute the *dissonance number* - the largest element with a catenary degree different from the fixed value. We determine the dissonance number in terms of the arithmetic sequence's starting point and its number of generators.

Gabriel Conant, University of Notre Dame
Title: VC-dimension in groups
Abstract: I will discuss recent work on the structure of VC-sets in groups, i.e. subsets whose family of left translates has absolutely bounded VC-dimension. Many tools from model theory and additive combinatorics can be adapted for "locally amenable" VC-sets, leading to stronger structural results than for arbitrary sets in (amenable) groups. These structure results can be applied to questions concerning sumset phenomena for VC-sets in infinite groups, as well as arithmetic regularity for VC-sets in finite groups.

Jeshu Dastidar, Queens College (CUNY)
Title: On the maximal number of roots of a trinomial over a prime field
Abstract: Canetti, Friedlander, et al. (2002) studied the randomness of powers over finite fields and along the way derived an analogue of Descartes rule over the finite field $\mathbb{F}_q$ with q elements: They showed that the number of roots of any univariate t-nomial, with exponents $\{0, a_2, \dots, a_t\}$ and the differences $a_i - a_j$ all relatively prime to $q-1$, is $O(q^{(t-2)/(t-1)})$. The correct optimal bounds remain a mystery

for prime fields, even in the case of polynomials with three terms. Following the work of Kelley (2016), we expand current evidence by using a supercomputer to determine the number of roots of these trinomials for $139,571 \leq p \leq 264,961$. We also prove that the search can be restricted to trinomials with a middle linear term when $p-1$ has less than three distinct prime factors.
This is joint work with Viviana Márquez and Ryan Pugh under the direction of Megan Ly and Dr. J. Maurice Rojas at the MSRI–UP REU in 2017.

Mauro Di Nasso, University of Pisa, Italy
Title: A generalized notion of asymptotic density
Abstract: Upper densities are monotone and subadditive functions from the power set of positive integers to the unit real interval that generalize the upper densities used in number theory, including the upper asymptotic density, the upper Banach density, and the upper logarithmic density. We answer a question posed by G. Grekos in 2013, and prove the existence of translation invariant abstract upper densities onto the unit interval, whose null sets are precisely the family of finite sets, or the family of sequences whose series of reciprocals converge. We also show that no such density can be atomless. (More generally, these results also hold for a large class of "summable" ideals.) This is a joint work with Renling Jin.

Robert Donley, Queensborough Community College (CUNY)
Title: Central zeros of Clebsch-Gordan Coefficients
Abstract: Clebsch-Gordan coefficients play a fundamental role in the representation theory of SU(2) and the quantum theory of electron coupling. An alternative approach is given using the binomial transform (Pascal's triangle). With the associated machinery (generating functions, recurrences, Regge symmetries), we give an elementary proof of a conjecture of S. Brudno, as clarified by Rau, and note several generalizations and applications.

Leonid Fel, Technion - Israel Institute of Technology
Title: Numerical semigroups generated by squares, cubes and quartics of three consecutive integers
Abstract: We derive the polynomial representations for minimal relations of generating set of numerical semigroups

$$R_n^k = \langle (n-1)^k, n^k, (n+1)^k \rangle, \qquad k = 2, 3, 4.$$

We find polynomial representations for degrees of syzygies in the Hilbert series $H(z; R_n^k)$ of these semigroups, their Frobenius numbers $F(R_n^k)$ and genera $G(R_n^k)$. We pose the conjecture about asymptotic behavior of $F(R_n^k)$ and $G(R_n^k)$ when $n \to \infty$ for arbitrary k.

Carrie Finch-Smith, Washington and Lee University
Title: Iterated Riesel and Sierpiński numbers
Abstract: A Riesel number is an odd positive integer k with the property that $k \cdot 2^n - 1$ is composite for all natural numbers n. In 1956, Hans Riesel showed that 509203 is an example of such a number, and he also showed that there are

infinitely many Riesel numbers. In this talk, we discuss iterating the process of multiplying by 2^n and subtracting 1 while maintaining compositeness. That is, we consider positive integers k with the property that $k \cdot 2^n - 1$, $(k \cdot 2^n - 1) \cdot 2^n - 1$, $\ldots$, $((\cdots(k \cdot 2^n - 1) \cdot 2^n - 1) \cdots) \cdot 2^n) - 1$ are all composite for all natural numbers. We also demonstrate analagous results for Sierpiński numbers.

Ayla Gafni, University of Rochester
Title: Additive energy and the metric Poissonian property
Abstract: Let A be a set of natural numbers. Recent work has suggested a strong link between the additive energy of A (the number of solutions to $a_1 + a_2 = a_3 + a_4$ with $a_i \in A$) and the metric Poissonian property, which is a fine-scale equidistribution property for dilates of A modulo 1. In particular, having "low" additive energy forces a set to be metric Poissonian, while "high" additive energy prevents the set from having this property. In this talk, I will discuss the history of the metric Poissonian property and its connection to additive energy. I will then present some results related to the intermediate range of energy, where other factors play a role in determining if a set has the metric Poissonian property.

Daniel Glasscock, Northeastern University
Title: Combinatorial cubes and solutions to linear equations in Piatetski-Shapiro sets
Abstract: It is a long-standing open problem to determine whether or not the set of squares contains a combinatorial cube with 7 elements, a configuration of the form $\{x, y, z, x+y, x+z, y+z, x+y+z\}$. In this talk, I will describe a metrical solution to the problem of finding such configurations in Piatetski-Shapiro sets, $\mathrm{PS}(\alpha) = \{\lfloor n^\alpha \rfloor \mid n \in \mathbb{N}\}$. Behind the scenes is an interesting dichotomy exhibited as the exponent α varies: for almost all small α, the set $\mathrm{PS}(\alpha)$ contains solutions to linear equations, while for almost all large α, the set $\mathrm{PS}(\alpha)$ does not.

Robert Hough, SUNY Stony Brook
Title: The local limit theorem on nilpotent Lie groups
Abstract: I will discuss new local limit theorems on the Heisenberg group, and on general connected, simply connected nilpotent Lie groups which extend the class of driving measures, generalizing earlier works of Breuillard and Alexopoulos. The techniques used include a permutation group action, application of the Gowers-Cauchy-Schwarz inequality, and the use of an approximate additive structure of the large spectrum of probability measures on additive groups. The work on the Heisenberg group is joint with Persi Diaconis.

Trevor Hyde, University of Michigan
Title: Polynomial factorization statistics and point configurations in $\mathbb{R}^3$
Abstract: Factorization statistics are functions defined on the set $\mathrm{Poly}_d(\mathbb{F}_q)$ of degree d polynomials with coefficients in $\mathbb{F}_q$ that only depend on the factorization type of a polynomial. We show there is a surprising connection between the expected values of factorization statistics on $\mathrm{Poly}_d(\mathbb{F}_q)$ and the structure of the cohomology of the space of point configurations in $\mathbb{R}^3$ as a symmetric group representation. This result forms a bridge between number theory and combinatorics on one hand, representation theory and topology on the other. We will demonstrate this interaction through examples if time permits.

Brad Isaacson, New York City Tech (CUNY)
Title: On some elementary character sums
Abstract: We discuss some character sums that can be evaluated using elementary techniques from algebra and number theory. As a corollary, we express a twisted generalization of the classical Dedekind sum as a linear combination of generalized Bernoulli numbers.

John H. Johnson, Ohio State University
Title: Upper Banach density along filters
Abstract: A recent observation of John Griesmer states that for upper Banach density $d^*(A) > \alpha$ if and only if for every finite nonempty $F \subseteq \mathbb{N}$ we have $\{t \in \mathbb{N} : |A \cap (F+t)| \geq \alpha \cdot |F|\} \neq \emptyset$. This observation makes precise the view that having positive upper Banach density is the same as being a "rarefied thick set". (A set is ***thick*** if and only if $\{t \in \mathbb{N} : |A \cap (F+t)| = |F|\} \neq \emptyset$ for every finite nonempty $F \subseteq \mathbb{N}$.) Generalizing this observation, we can define a notion of upper Banach density relative to filters on $(\mathbb{N}, +)$. We show that, under certain conditions, this "filtered" notion of density still shares many of the same combinatorial properties as the classical notion of upper Banach density such as translation invariance, partition regularity, and recurrence. Joint work with Florian Richter.

Mizan R. Khan, Eastern Connecticut State University
Title: Hensley's Theorem on clean lattice tetrahedra with interior lattice points
Abstract: An empty lattice tetrahedron is a tetrahedron in $\mathbb{R}^3$ whose vertices are lattice points, but does not contain any other lattice points. The typical example is the tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0)$ and $(1,1,n)$ where n is any non-zero integer. Since n is unbounded, this shows that there is no bound on the volume of empty tetrahedra. A clean tetrahedra is a lattice tetrahedron whose sides do not contain any lattice points other than the vertices. (It is permissible for lattice points to lie in the interior.) An example of a clean lattice tetrahedron which is not an empty lattice tetrahedron is the tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0)$ and $(2,3,11)$. It is natural to extrapolate from the empty tetrahedra case that clean lattice tetrahedra containing a fixed non-zero number of lattice points in the interior have unbounded volume. In the 80's Hensley proved that this was untrue. He gave an inductive proof of the fact that the

set of clean tetrahedra containing a fixed non-zero number of lattice in the interior have bounded volume. We will describe the geometric idea underlying the proof.

Angel Kumchev, Towson State University
Title: A hybrid of two theorems of Piatetski-Shapiro
Abstract: The Piatetski-Shapiro primes are a classical example of a thin sequence of primes that has served as a test case in many investigations on additive problems in thin sets of primes. Another question about primes first posed by Piatetski-Shapiro deals with the solubility in primes $p_1, \dots, p_s$ of the Diophantine inequality

$$|p_1^c + \cdots + p_s^c N| < \epsilon.$$

Here, $\epsilon > 0$ is small, $c > 1$ is non-integer, and N is a large parameter. This version of the Waring-Goldbach problem for non-integer exponents has attracted a lot of attention during the last quarter of a century. In this talk, I will present some joint work with Zh. Petrov (Sofia University) on the solubility of the above Diophantine inequality in Piatetski-Shapiro primes.

Kevin Kwan, Columbia University, and **Steven J. Miller**, Williams College
Title: Near-perfect, within-perfect, and order-a-abundant numbers
Abstract: A perfect number n equals the sum of its proper divisors, or $\sigma(n) = 2n$ where σ is the sum of divisors function. Perfect numbers have been studied since Euclid, but many ancient questions, such as the infinitude of perfect numbers and the existence of odd perfect numbers, remain wide open. Building on the analytic techniques of Erdős, Pollack, and Pomerance, we study two types of approximate perfect numbers. Given a non-negative function k, we say a number n is *k-within-perfect* if $|\sigma(n) - 2n| < k$, and we say n is *k-near-perfect* if n can be written as the sum of all but at most k of its proper divisors. We prove the existence of a phase transition in the density of k-within-perfect numbers as k varies from sub-linear to super-linear and establish some results on their growth rate. This generalizes previous results, which were limited to the case where k is constant. If time permits we will mention some related results on generalizations of abundant numbers.

Steven Leth, University of Northern Colorado
Title: Some questions and results about SIM sets
Abstract: In this talk I will discuss recent joint work with Isaac Goldbring involving the use of nonstandard methods and "SIM sets." These are standard sets with a very natural measure property in the non-standard setting. There exist SIM sets that have zero Banach Density, but a long-standing open question asks whether every set of positive Banach density contains a SIM set. With the recent resolution of the question of the existence of sumsets in sets of positive Banach density by Moreira, Richter and Robertson, it is natural to ask whether the same result is true for SIM sets. We make some modest progress towards this goal, and also give improved characterizations of SIM sets.

Huixi Li, Clemson University
Title: On Lemoine's conjecture and a theorem by Estermann
Abstract: In 1931 Estermann proved that every sufficiently large integer is a sum of a prime and a square-free number. If Goldbach's conjecture on even numbers and Lemoine's conjecture on odd numbers are both true, then we know every integer greater than 3 is a sum of a prime and a square-free number with at most two prime divisors. In this presentation I will talk about the result that every sufficiently large odd integer is a sum of a prime and a square-free integer with at most three prime divisors. Then together with Chen's theorem, we know every sufficiently large integer is a sum of a prime and a square-free number with at most three prime divisors.

Jared Duker Lichtman, Dartmouth College
Title: Explicit estimates for the distribution of numbers free of large prime factors
Abstract: A number is called y-smooth if all its prime factors are at most y. There is a large literature on the distribution of y-smooth numbers up to x. Asymptotic estimates for this distribution are quite useful in many applications, including the analysis of factorization and discrete logarithm algorithms. However, very little is known about this distribution that is numerically explicit. We generalize the saddle-point method of Hildebrand and Tenenbaum, giving explicit and fairly tight intervals in which the true count lies. For example, we determine that the number of 500-digit integers free of $\geq$ 35-digit prime factors lies within 0.65% of 1.505×10^{482}. This interval is so tight we can exclude the famous Dickmande Bruijn asymptotic estimate as too small (1.472×10^{482}) and the HildebrandTenenbaum main term as too large (1.513×10^{482}). One of the essential ideas in the proof is to relate the desired count of smooth numbers to a contour integral over a vertical line in the complex plane. The integrand includes an oscillatory sum over the primes, which requires an explicit form of the Prime Number Theorem. Joint work with Carl Pomerance.

Diego Marques, Universidade de Brasília, Brazil
Title: On some problems proposed by Kurt Mahler
Abstract: In this lecture, we will present some problems proposed by K. Mahler and related to the arithmetic behavior of transcendental functions. Moreover, we shall provide the recent advances on this topic.

Ariane Masuda, New York City Tech (CUNY)
Title: Permutation polynomials over $\mathbb{F}_{q^2}$ from rational functions
Abstract: Let $\mathbb{F}_q$ be the finite field of q elements. A polynomial f in $\mathbb{F}_q[x]$ is a *permutation polynomial* over $\mathbb{F}_q$ if the induced mapping $x \mapsto f(x)$ is a bijection in $\mathbb{F}_q$. We discuss a method for constructing permutation polynomials over $\mathbb{F}_{q^2}$ by using rational functions that induce bijections either on the set μ_{q+1} of the $(q+1)$-th roots of unity or between μ_{q+1} and $\mathbb{F}_q \cup \{\infty\}$. This is joint work with Daniele Bartoli and Luciane Quoos.

Azita Mayeli, Queensborough CC and CUNY Graduate Center
Title: Existence of no Gabor orthogonal bases on the smooth and symmetric convex sets
Abstract: Let K be a convex and symmetric bounded set in $\mathbb{R}^d$, $d \geq 2$, with smooth boundary. Using a combinatorial approach, in this talk we show that for $d \neq 1(mod4)$, the indicator function of K can not serve as an orthogonal Gabor window function for $L^2(\mathbb{R}^d)$, i.e., there is no countable set $S \subset \mathbb{R}^{2d}$ such that the Gabor family $\mathcal{G}(1_K, S) = \{e^{2\pi ix\cdot b}1_K(x-a) : (a,b) \in S\}$ is an orthogonal basis for $L^2(\mathbb{R}^d)$. This is joint work with Alex Iosevich.

Katie McKeon, Rutgers University
Title: Low-lying fundamental geodesics in an arithmetic hyperbolic 3-manifold
Abstract: We'll examine closed geodesics in the quotient of hyperbolic three space by the discrete group of isometries SL(2,Z[i]). There is a correspondence between closed geodesics in the manifold, the complex continued fractions originally studied by Hurwitz, and binary quadratic forms over the Gaussian integers. According to this correspondence, a geodesic is called fundamental i f the associated binary quadratic form is. Using techniques from sieve theory, symbolic dynamics, and the theory of expander graphs, we show the existence of a compact set in the manifold containing infinitely many fundamental geodesics.

Nathan McNew, Towson University
Title: Counting primitive subsets of $\{1, 2, ...n\}$
Abstract: Let $f(n)$ count the number of primitive subsets of the integers $\{1, 2, ...n\}$, where primitive means that no integer in the set divides another integer in the set. In 1988 Cameron and Erdős noted that $2^n > f(n) > 2^{\lfloor \frac{n}{2} \rfloor} \approx (\sqrt{2})^n$, which follows from the fact that any subset of the integers from $\lceil n/2 \rceil$ to n is primitive. They conjectured that in fact $f(n)$ was asymptotic to c^n for some constant c, and showed that $1.55^n < f(n) < 1.59^n$. In 2017 this conjecture was proven by Angelo, who showed that $f(n) \sim c^n$ for some c, but the proof is ineffective in the sense that it gives no information about the value of the constant c.

We give an alternate proof of this result which gives more than just an asymptotic for $f(n)$ and also shows that the constant c is effectively computable. For example we can improve the bounds to $1.572 < c < 1.574$. We also show that the same method can be used to count subsets which are geometric-progression-free.

Joel Moreira, Northwestern University
Title: A solution to the Erdős sumset conjecture
Abstract: Erdős conjectured that every subset A of the natural numbers with positive density contains a sumset $B + C := \{b + c : b \in B, c \in C\}$ for infinite sets B and C of natural numbers. I will present a solution of this conjecture, obtained recently in joint work with Richter and Robertson. There are three main steps. The first is to rewrite the statement using functions instead of sets; we do this using the language of ultrafilters. The second is to decompose (the indicator function of) a set with positive density into an "almost periodic" component and a "pseudo-random" component. In fact, we need two distinct such decompositions.

Finally we tie everything together, using some ideas and tools from ergodic theory. No knowledge of ergodic theory or ultrafilters will be assumed.

Mel Nathanson, CUNY (Lehman College and the Graduate Center)
Title: Explicit computation of Sinkhorn limits
Abstract: Let A be an $n \times n$ positive matrix, and let $(A_k)_{k=1}^{\infty}$ be the sequence of positive matrices obtained by alternately column scaling and row scaling. Sinkhorn's theorem asserts that the sequence $(A_k)_{k=1}^{\infty}$ converges to a doubly stochastic matrix. Explicit limits are computed for 3×3 symmetric matrices.

Kevin O'Bryant, College of Staten Island (CUNY)
Title: Explicit bounds for primes in arithmetic progressions
Abstract: We present fully explicit bounds on the number of primes less than x and congruent to a mod q, valid for all a, q, and modestly sized x. Joint work with Mike Bennett, Greg Martin, and Andrew Rechnitzer.

Péter Pál Pach,
Title: On multiplicative Sidon sets
Abstract: While the topic of additive Sidon sets is well-studied in additive number theory, much less attention has been devoted to the multiplicative case. In this talk we will discuss results about multiplicative Sidon sets.

Ram Krishna Pandey, Indian Institute of Technology Roorkee, India
Title: Direct and inverse theorems on the general h-fold sumsets and on the sums of dilated sets
Abstract: Let A be a nonempty finite set of integers and h and r be positive integers. The generalized h-fold sumset, denoted by $h^{(r)}A$, is the sum of h elements of A, where each element appears in the sum can have at most r repetitions. Further, the Minkowski sum of the dilated sets is defined by

$$m_1 \cdot A + m_2 \cdot A + \cdots + m_k \cdot A = \{m_1 a_1 + m_2 a_2 + \cdots + m_k a_k ; a_i \in A \text{ for } 1 \leq i \leq k\}.$$

Here, $m_1, m_2, \ldots, m_k$ are positive integers with $\gcd(m_1, m_2, \ldots, m_k) = 1$. In this talk, we present some direct and inverse results for $|h^{(r)}A|$ and $|m_1 \cdot A + m_2 \cdot A + \cdots + m_k \cdot A|$.

Cosmin Pohoata, California Institute of Technology
Title: Higher energies in additive combinatorics and discrete geometry
Abstract: Using geometric variants of additive higher moment energies (introduced by Schoen and Shkredov), we discuss improved bounds for the problem of distinct distances with local properties and new bounds for problems involving expanding polynomials in $\mathbb{R}[x, y]$ when one or two of the sets have structure.

Yuneid Puig de Dios, University of California, Riverside
Title: Recurrence sets of linear operators with high piecewise syndeticity level.
Abstract: Our talk is concerned with the interplay of dynamics of linear operators and combinatorial number theory. We study the dynamics of linear and bounded operators with recurrence sets having a high piecewise syndeticity level, and show a result motivated by a question posed by Sophie Grivaux concerning the regularity of the orbits of frequently hypercyclic operators.

Alex Rice, Millsaps College
Title: Extending the best known bounds for the Furstenberg-Sárközy Theorem
Abstract: It is a well known result, established independently by Sárközy and Furstenberg, that a set of integers with positive upper density must contain two distinct elements that differ by a perfect square. The best-known quantitative upper bounds for this result were established with an intricate Fourier analytic argument by Pintz, Steiger, and Szemerédi. In this talk, we discuss the extension of these bounds from perfect squares to, at long last, the largest possible class of polynomials, as well as other related results.

David Ross, University of Hawaii
Title: Non-archimedean extensions of $\mathbb{R}$ and diophantine equations connected to Egyptian fractions
Abstract: Egyptian fractions are closely related to a variety of interesting Diophantine equations, such as the Kellogg Equation ($1/x_1 + 1/x_2 + \cdots + 1/x_s = 1$), the Znám Equation ($1/x_1 + \cdots + 1/x_s + 1/x_1x_2 \cdots x_s = a$), and the Lagarias Equation ($c(1/x_1 + \cdots + 1/x_s) + b/x_1x_2 \cdots x_s = a$). I'll discuss the way certain non-Archimedean extensions of $\mathbb{R}$ can give insight into the structure of the solution sets of such equations, and therefore into the set of numbers representable as Egyptian fractions. For example, I'll show that sets such as these are generally closed, extending an old result of Sierpinski.

Eric Rowland, Hofstra University
Title: Enumeration of binomial coefficients by their p-adic valuations
Abstract: In 1947 Nathan Fine obtained a beautiful formula for the number of binomial coefficients $\binom{n}{m}$, for fixed n, that are not divisible by p: if the standard base-p representation of n is $n_\ell \cdots n_1 n_0$, then this number is

$$(n_0 + 1)(n_1 + 1) \cdots (n_\ell + 1).$$

Subsequently, many authors found formulas counting binomial coefficients with p-adic valuation $\nu_p(\binom{n}{m}) = \alpha$ for particular values of p and α, but a general formula remained elusive. We give a matrix product, generalizing Fine's result, for the generating function

$$T_p(n, x) := \sum_{m=0}^{n} x^{\nu_p(\binom{n}{m})}$$

which simultaneously counts binomial coefficients with p-adic valuation α for all $\alpha \geq 0$. Namely, if

$$M_p(d) := \begin{bmatrix} d+1 & p-d-1 \\ d\,x & (p-d)\,x \end{bmatrix}$$

then

$$T_p(n,x) = \begin{bmatrix} 1 & 0 \end{bmatrix} M_p(n_0)\, M_p(n_1) \cdots M_p(n_\ell) \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

A further generalization, involving $k \times k$ matrices, counts multinomial coefficients by their p-adic valuations.

Andrey Rukhin, NSWCDD. Dept. Of Navy
Title: Alternative proofs of Steiner's 1-cycle theorem
Abstract: Within the context of the $3x+1$ Problem, Steiner's *1-Cycle Theorem* is a result pertaining to the non-existence of certain periodic orbits (known as *circuits*) within the $3x+1$ dynamical system. For all $a, b \in \mathbb{N}$, Steiner demonstrated that rational expressions of the form

$$\frac{2^a - 1}{2^{a+b} - 3^b}$$

do not assume a positive integer value, except in the case where $a = b = 1$ (corresponding to the $(1, 4, 2)$ orbit); he does so by appealing to the continued fraction expression of $\log_2 3$, transcendental number theory, and numerical computation.

In this talk, the speaker offers alternative proofs of Steiner's theorem from a algebraic perspective. Assuming an upper bound on periodic iterates that was established by Belaga and Mignotte (appealing to the work of Baker and Wüstholz), these proofs employ elementary identities from modular arithmetic, weighted binomial coefficients, and the *graded* 2-*adic* and 3-adic expansions of rational expressions such as above.

John R. Schmitt, Middlebury College
Title: Distinct Partial Sums in Cyclic Groups
We consider some simply stated problems and conjectures arising from the study of combinatorial designs. These problems may be broadly described as follows: a finite subset of the elements of some group is given and one wishes to order the elements of this finite subset so that the sequence of partial sums has terms that are distinct. Within this setting there are a plethora of questions that one might consider; these arise as one varies the group, places restrictions on the elements of the subset chosen, or imposes additional restrictions upon the sequence of partial sums beyond the terms being distinct.

To be specific, let $(G, +)$ be an abelian group and consider a subset $A \subseteq G$ with $|A| = k$. Given an ordering $(a_1, \ldots, a_k)$ of the elements of A, define its *partial sums* by $s_0 = 0$ and $s_j = \sum_{i=1}^{j} a_i$ for $1 \leq j \leq k$.

We will consider the following conjecture attributed to Alspach: For any cyclic group $\mathbf{Z}_n$ and any subset $A \subseteq \mathbf{Z}_n \setminus \{0\}$ with $s_k \neq 0$, it is possible to find an ordering of the elements of A such that no two of its partial sums s_i and s_j are equal for $0 \leq i < j \leq k$.

We also consider a weakening of this conjecture due to Archdeacon.

We address these conjectures in the case that n is *prime* and do the following. We show how Noga Alon's Combinatorial Nullstellensatz can be used to frame these conjectures. Further, in the case that n is prime, we verify computationally that the conjecture is true for small values of $|A|$. In the case that n is prime, we show that a sequence of length k having distinct partial sums exists in any subset of $\mathbf{Z}_n \setminus \{0\}$ of size at least $2k - \sqrt{8k}$ in all but at most a bounded number of cases. Joint work with Jacob Hicks and Matt A. Ollis.

Steven Senger, Missouri State University
Title: Rainbow point configurations
Abstract: We prove some very general results about which colorings of sets guarantee special subsets with each element from a different color class. We present many examples such as point configurations in various vector spaces, as well as some additive combinatorial type results in abelian groups.

Arseniy Sheydvasser, Yale University
Title: Rigidity in the Ulam sequence
Abstract: Let $U(a, b)$ denote the generalized Ulam sequence starting with integers a, b, such that each subsequent term is the smallest integer that can be written as the sum of two distinct preceding terms in exactly one way. Ulam, for which the sequence is named, introduced $U(1, 2)$ in 1964. Since then, it has continued to engender both interest and frustration due to appearing to satisfy clear patterns that nevertheless continue to be impossible to prove. In this talk, we shall discuss some more recent results about families of Ulam sequences. Our general theme is that while any individual Ulam sequence is chaotic, families of Ulam sequences are surprisingly rigid. Along the way, we shall discuss both some of the uses and limitations of model theory in studying this problem.

Satyanand Singh, New York City Tech (CUNY)
Title: A Refinement of the Calkin Wilf Tree with some of its Implications
Abstract: We study a refinement of the Calkin-Wilf tree due to Nathanson in this presentation. In particular, we consider the properties of such trees associated with the matrices $L_u = \begin{bmatrix} 1 & u \\ 0 & 1 \end{bmatrix}$ and $R_v = \begin{bmatrix} 1 & 0 \\ v & 1 \end{bmatrix}$, where u and v are nonnegative integers. We extend several known results of the original Calkin-Wilf tree, including symmetry, numerator-denominator, and successor formulas, to this new setting. The talk will columinate with some recent results on maximal entries in certain matrix monoids. This is based on joint work with Sandie Han, Ariane M. Masuda and Johann Thiel.

Christoph Spiegel, Universitat Politécnica de Catalunya, Spain
Title: Going beyond 2.4 in Freiman's 2.4-Theorem
Abstract: Freiman's 2.4-Theorem states that any set $A \subset \mathbf{Z}_p$ satisfying $|2A| \leq 2.4|A| - 3$ and $|A| < p/35$ can be covered by an arithmetic progression of length at most $|2A| - |A| + 1$. A more general result of Green and Ruzsa implies that this covering property holds for any set satisfying $|2A| \leq 3|A| - 4$ as long as the rather

strong density requirement $|A| < p/10^{215}$ is satisfied. We present a version of this statement that allows for sets satisfying $|2A| \leq 2.48|A| - 7$ with the more modest density requirement of $|A| < p/10^{10}$.
This is joint work with Pablo Candela and Oriol Serra.

Wenbo Sun, Ohio State University
Title: Solution counting problem and quantitative recurrence theorem
Abstract: Let $p(x_1, \dots, x_n)$ be a linear function with n variables. In combinatorics, the solution counting problem studies the number of solutions for the equation $p(x_1, \dots, x_n) = 0$ for $x_1, \dots, x_n$ lying in a given subset of integers. In ergodic theory, the quantitative recurrence theorem studies the lower bound for the measure of the set $A \cap T^{a_1 n} A \cap \cdots \cap T^{a_d n} A$ over a large set of return time n for some integers $a_1, \dots, a_d$. In this talk, I will talk about recent advances for these topics, and discuss the connection between these two questions.
This is joint work with Sebastian Donoso, Anh Leeand Joel Moreira.

Caroline Amelia Terry, University of Maryland
Title: A stable arithmetic regularity lemma in finite abelian groups
Abstract: The arithmetic regularity lemma for $\mathbb{F}_p^n$ (first proved by Green in 2005) states that given $A \subseteq \mathbf{F}_p^n$, there exists $H \leq \mathbf{F}_p^n$ of bounded index such that A is Fourier-uniform with respect to almost all cosets of H. In general, the growth of the index of H is required to be of tower type depending on the degree of uniformity, and must also allow for a small number of non-uniform elements. Previously, in joint work with Wolf, we showed that under a natural stability theoretic assumption, the bad bounds and non-uniform elements are not necessary. In this talk, we present results extending these results to stable subsets of arbitrary finite abelian groups.
This is joint work with Julia Wolf.

Lola Thompson, Oberlin College
Title: Divisor-sum fibers
Abstract: Let $s(\cdot)$ denote the sum-of-proper-divisors function, that is, $s(n) = \sum_{d|n,\ d<n} d$. Erdős–Granville–Pomerance–Spiro conjectured that, for any set $\mathcal{A}$ of asymptotic density zero, the preimage set $s^{-1}(\mathcal{A})$ also has density zero. We prove a weak form of this conjecture. In particular, we show that the EGPS conjecture holds for infinite sets with counting function $O(x^{\frac{1}{2}+\epsilon(x)})$. We also disprove a hypothesis from the same paper of EGPS by showing that for any positive numbers α and ϵ, there are integers n with arbitrarily many s-preimages lying between $\alpha(1-\epsilon)n$ and $\alpha(1+\epsilon)n$. This talk is based on joint work with Paul Pollack and Carl Pomerance.

Henry Towsner, University of Pennsylvania
Title: Generalizing VC dimension to higher arity
Abstract: The notion of bounded VC dimension is a property at the intersection of combinatorics and probability. This family has been discovered repeatedly and studied from various perspectives - for instance, in model theory, theories with

bounded VC dimension are known as NIP (the theories which do Not have the Independence Property). One useful property is that graphs with bounded VC dimension are the graphs that can be always be finitely approximated in a "random-free" way: graphs with bounded VC dimension satisfy a strengthening of Szemeredi's Regularity Lemma in which the densities between the pieces of the partition are either close to 0 or close to 1. The generalization of VC dimension to higher arity, known in model theory as k-NIP for various k, has been less well-studied. We summarize some known facts about this generalization, including a new result (joint with Chernikov) showing k-NIP hypergraphs have a similar kind of approximation with only "lower order" randomness.

Yuri Tschinkel, Courant Institute, New York University
Title: Arithmetic of del Pezzo surfaces
Abstract: I will survey recent advances on problems of rationality and rational points on del Pezzo surfaces.

Maximilian Wötzel, Universitat Politècnica de Catalunya, Spain
Title: Set systems with distinct sumsets
Abstract: A family $\mathcal{A}$ of k-subsets of $\{1, 2, \ldots, N\}$ is a Sidon system if the sumsets $A + A'$, $A, A' \in \mathcal{A}$ are pairwise distinct. We show that the largest cardinality $F_k(N)$ of a Sidon system of k-subsets of $[N]$ satisfies $F_k(N) \leq \binom{N-1}{k-1} + N - k$ and the asymptotic lower bound $F_k(N) = \Omega_k(N^{k-1})$. More precise bounds on $F_k(N)$ are obtained for $k \leq 3$. We also obtain the threshold probability for a random system to be Sidon for $k \leq 3$. This is joint work with Javier Cilleruelo and Oriol Serra.