CANT 2019 Abstracts

Seventeenth Annual Workshop on Combinatorial and Additive Number Theory

CUNY Graduate Center
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Paul Baginski, Fairfield University
Title: Abundant numbers and nonunique factorization
Abstract: An integer n is abundant if the sum of its divisors, $\sigma(n) = \sum_{d|n} d$,
is greater than $2n$; n is perfect if $\sigma(n) = 2n$; and otherwise n is deficient. Both the set H of abundant numbers and the set H^* of non-deficient numbers are closed under multiplication, making them submonoids of $(\mathbb{N}, \times)$. Both of these monoids have elements with nonunique factorization into irreducibles. We will discuss the factorization properties these monoids possess and connections to known problems about abundant numbers.
Joint work with Jill Stifano.

Amanda Burcroff, University of Michigan
Title: Covering problems involving the unitary Cayley graph of $\mathbb{Z}/n\mathbb{Z}$
Abstract: Let C_n denote the set of integers relatively prime to n, and let $\gamma_t(n)$ denote the minimum number of translates of C_n needed to cover $\mathbb{Z}$. We answer a question of Defant and Iyer by constructing a family of infinitely many integers n such that $\gamma_t(n) \leq g(n) - 2$, where $g(n)$ is the Jacobsthal function, the minimum m such that every set of m consecutive integers contains an integer coprime to n. The unitary Cayley graph of $\mathbb{Z}/n\mathbb{Z}$, denoted X_n, is the graph on $\{0, \dots, n-1\}$ where vertices a and b are adjacent if $a - b$ is coprime to n. The parameter $\gamma_t(n)$ can be viewed graph-theoretically as the total domination number of X_n.

When n is squarefree, we bound the minimum number of translates of $C_n \cup \{0\}$ needed to cover $\mathbb{Z}$ and the minimum number of translates of C_n by elements of C_n needed to cover $\mathbb{Z}$. These correspond to the domination number and lower independence number of X_n, respectively. When n has a bounded smallest prime factor, we also give asymptotic bounds for the maximum number of translates of C_n minimally covering an arbitrary subset of $\mathbb{Z}$, i.e., the upper irredundance number of X_n.

Colin Defant, Princeton University
Title: Connected components of ranges of divisor functions
Abstract: For each complex number c, the divisor function σ_c is defined by $\sigma_c(n) = \sum_{d|n} d^c$. This talk considers the topological properties of $\overline{\sigma_c(\mathbb{N})}$, the closure of the range of σ_c. More precisely, we will discuss $\mathcal{N}(c)$, the number of connected components of $\overline{\sigma_c(\mathbb{N})}$. There have been several recent developments toward understanding $\mathcal{N}(c)$ when c is a negative real number. In this setting, Sanna proved that $\mathcal{N}(c)$ is finite, and the combined efforts of Zubrilina, Achenjang, and Berger have yielded

asymptotic estimates for these numbers. We will also see that $\mathcal{N}(c)$ is finite whenever c is a complex number with negative real part. The second half of the talk concerns some wide open problems, including the following question. For which bounded multiplicative functions $f : \mathbb{N} \to \mathbb{C}$ is it the case that $\overline{f(\mathbb{N})}$ has finitely many connected components?

Robert W. Donley, Jr., Queensborough Community College (CUNY)
Title: The gentle art of 3×3 semi-magic squares
Abstract: In 1958, the physicist Tullio Regge made the remarkable observation that the domain space for electron coupling calculations (Clebsch-Gordan coefficients) consists of 3×3 semi-magic squares and that such computations transform under the full group of 72 determinantal symmetries on 3×3 matrices. Here "semi-magic" means that entries are nonnegative integers and that sums along rows and columns are constant. To visualize the five-dimensional domain space, we propose a three-dimensional model whose points represent hexagonal families of semi-magic squares. In turn, the vanishing points of the Clebsch-Gordan function produce portraits in this model with intriguing artistic qualities.

Heidi Goodson, Brooklyn College (CUNY)
Title: Vertically aligned entries in Pascal's triangle and applications to number theory
Abstract: The classic way to write down Pascal's triangle leads to entries in alternating rows being vertically aligned. In this talk, I will give a linear dependence on vertically aligned entries in Pascal's triangle. Furthermore, I will give an application of this dependence to number theory. Specifically, I will explain how a search for morphisms between hyperelliptic curves led to the discovery of this identity.

David Grynkiewicz, University of Memphis
Title: An inverse zero-sum problem for elementary abelian p-groups of rank two
Abstract: Let $G = \mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$ be an elementary abelian p-group of rank two. For an integer $k \geq 1$, let $\mathsf{s}_{\leq k}(G)$ denote the minimal integer $\ell \geq 1$ such that any sequence of terms from G having length at least ℓ must contain a nonempty zero-sum subsequence of length at most k. It is trivial that $\mathsf{s}_k(G) = \infty$ for $k < p$, and that $\mathsf{s}_{\leq k}(G) = \mathsf{s}_{\leq \mathsf{D}(G)}(G)$ for any $k \geq \mathsf{D}(G)$, where $\mathsf{D}(G) = 2p - 1$ denotes the Davenport constant of G, which is the minimal integer ℓ such that any sequence of length ℓ must contain a nonempty zero-sum subsequence of some length. The range of interest is then $k \in [p, 2p-1]$. The value of $\mathsf{s}_{\leq k}(G)$ for the extremal values $k = p$ and $k = 2p - 1$ are classical, and go back to the work of Olson. More recent work of Wang and Zhao has determined that $\mathsf{s}_{\leq k}(G) = 4p - 2 - k$ for any $k \in [p, 2p - 1]$.

In this talk, we are concerned with the related inverse problem for $\mathsf{s}_{\leq k}(G)$. Namely, we wish to completely characterize all sequences S of maximal possible length $\mathsf{s}_{\leq k}(G) - 1 = 4p - 3 - k$ that *fail* to have the desired nonempty zero-sum subsequence of length at most k. When $k = 2p - 2$, the problem reduces to characterizing all zero-sum sequences of length $2p - 1$ having no proper, nonempty zero-sum subsequence. A major achievement of Reiher established the conjectured structure of such sequences (known in the literature as Property B) for all primes

$p \geq 2$. The case $k = 2p - 1$ follows directly from the case $k = 2p - 2$, and the characterization for $k = p$, known in the literature as Property C, had also been shown by a more involved argument (of Gao and Geroldinger) to reduce to the case $k = 2p - 2$, thus following from the work of Reiher as well.

In this talk, we present work characterizing those extremal sequences S when k lies in the broader range $k \in [\frac{4}{3}(p-1), 2(p-1)]$. Specifically, we show that if S is a sequence of $4p - 3 - k$ terms from G containing no nonempty zero-sum subsequence of length at most k, then there must some basis (e_1, e_2) of G such that S consists of the terms e_1 and e_2, each with multiplicity $p - 1$, as well as the term $e_1 + e_2$ with multiplicity $p - 1 - k$. In other words, if we write the sequence S as a multiplicative string using $g^{[y]}$ to denote the term $g \in G$ repeated $y \geq 0$ times, S must have the following form:
$$S = e_1^{[p-1]} \cdot e_2^{[p-1]} \cdot (e_1 + e_2)^{[p-1-k]}.$$

In contrast to the extremal values $k \geq 2p - 2$ and $k = p$, this shows more regular structure holds outside the extremal values for k.

Joint work with Chunlin Wang and Kevin Zhao.

Matthew Hase-Liu, Harvard University
Title: Sum-product phenomena for planar hypercomplex numbers
Abstract: We study the sum-product problem for the planar hypercomplex numbers: the dual numbers and double numbers. These number systems are similar to the complex numbers, but it turns out that they have a very different combinatorial behavior. We identify parameters that control the behavior of these problems, and derive sum-product bounds that depend on these parameters. For the dual numbers we expose a range where the minimum value of $\max\{|A + A|, |AA|\}$ is neither close to $|A|$ nor to $|A|^2$. To this end, we study point-line incidences in the dual plane and in the double plane, developing analogs of the Szemerédi-Trotter theorem and extensions of Elekes' sum-product technique. As in the case of the sum-product problem, it turns out that the dual and double variants behave differently than the complex and real ones.

Charles Helou, Penn State, Brandywine
Title: A characterization of representation functions
Abstract: Given a set A of natural numbers, i.e., nonnegative integers, the representation function of A is the function which to every natural number n associates the number $r_A(n)$ of ordered pairs (a, b) of elements $a, b \in A$ such that $a + b = n$. We establish an intrinsic characterization of representation functions, thus answering an open problem stated many years ago by Mel Nathanson.

Robert Hough, SUNY - Stony Brook Title: Cut-off phenomenon for the abelian sandpile model on tiling graphs
Abstract: In the abelian sandpile model on a graph $G = (V, E)$ with sink s, there are a non-negative number of chips $\sigma(v)$ at each non-sink vertex v. If $\sigma(v) \geq \deg(v)$, the vertex can 'topple' passing one chip to each neighbor. Any chips which fall on the sink are lost from the model. A configuration σ is called 'stable' if no vertex can topple. In driven dynamics in the model, at each step a chip is added

to the model at a uniform random vertex, and all legal topplings are performed until a stable configuration is reached. Together with Dan Jerison and Lionel Levine, I determined the asymptotic mixing time to stationarity and proved a cut-off phenomenon for dynamics on a square $N \times N$ grid with periodic boundary conditions and a single sink in the limit $N \to \infty$. Recently, with Hyojeong Son, I have extended this result to prove a cut-off phenomenon for sandpile dynamics on a growing piece of an arbitrary plane or space tiling, with open or periodic boundary condition, and proved that the asymptotic mixing time is equal in two dimensions subject to a reflection condition. A different boundary behavior exists for the D4 lattice in dimension 4, in which the open boundary can change the mixing time. The methods include an analysis of the functions which are 'harmonic modulo 1' on a tiling graph and of the asymptotic behavior of the tiling's Green's function.

Jing-Jing Huang, University of Nevada, Reno
Title: The rational points close to a manifold
Abstract: We will mainly talk about problems related to counting rational points in a thin neighborhood of a manifold. We have recently solved the case of hypersurfaces, and made some progress in the case of space curves. There are also significant applications of our results to diophantine inequalities and metric diophantine approximation on manifolds.

Trevor Hyde, University of Michigan
Title: Cyclotomic factors of necklace polynomials
Abstract: Necklace polynomials enumerate aperiodic necklaces of colored beads. They have a long history passing through number theory, geometry, representation theory, and combinatorics. I will discuss some recent work which began with the observation that necklace polynomials vanish at many roots of unity and eventually led to connections with multiplicative cyclotomic unit relations and the Euler characteristics of the space of all multivariate irreducible polynomials.

Alex Iosevich, Univeristy of Rochester
Title: On certain analytic, combinatorial and number-theoretic aspects of frame theory
Abstract: We are going to describe some recent results pertaining to the existence and non-existence of exponential bases and frames and Gabor bases. We shall emphasize the plethora of techniques and ideas from several areas of mathematics that go into the analysis of the underlying phenomena.

Mizan R. Khan, Eastern Connecticut State University
Title: A conjecture on visible points in lattice parallelograms
Abstract: Let $a, n \in \mathbb{Z}^+$, with $a < n$ and $\gcd(a, n) = 1$. Let $P_{a,n}$ denote the lattice parallelogram spanned by $(1, 0)$ and (a, n), that is,

$$P_{a,n} = \{t_1(1, 0) + t_2(a, n) : 0 \le t_1, t_2 \le 1\},$$

and let $V(a, n)$ be the number of visible lattice points lying in the interior of $P_{a,n}$. We will state some elementary (and straightforward) results for $V(a, n)$. We conjecture that for $a \neq 1, n-1$, $V(a, n)$ satisfies the inequality

$$0.5n < V(a, n) < 0.75n.$$

The basic heuristic for the upper bound is that at least one-fourth of the lattice points inside the parallelogram have both even x and y co-ordinates. However (other than extensive computer calculations) we do not have a heuristic for the lower bound.
Joint work with Joydip Saha and Peng Zhao.

Sandor Kiss, Budapest University of Technology and Economics, Hungary
Title: A problem of Erdős about sets without pairwise coprime integers
Abstract: Let $f(n, k)$ be the largest cardinality of the subsets of positive integers not exceeding n which does not contain $k+1$ pairwise coprime integers. Let $E(n, k)$ be the set of positive integers up to n which have least one prime divisor among the first k primes. Erdős conjectured in 1962 that $f(n, k)$ is equal to the cardinality of $E(n, k)$ for all large enough n. In my talk I am telling some of our recent results about the conjecture.

Alex Kontorovich, Rutgers University, New Brunswick
Title: On working with Jean Bourgain
Abstract: We will give a personal/historical overview of some problems of interest to Bourgain, and recent developments thereof.

Ben Krause, California Institute of Technology
Title: Discrete analogues in harmonic analysis: Directional maximal functions in $\mathbb{Z}^2$
Abstract: Let $V = \{v_1, \ldots, v_N\}$ be a collection of N vectors that live near a discrete sphere. We consider discrete directional maximal functions on $\mathbb{Z}^2$ where the set of directions lies in V, given by

$$\sup_{v \in V, k \geq C \log N} \left| \sum_{n \in \mathbb{Z}} f(x - v \cdot n) \cdot \phi_k(n) \right|, \ f : \mathbb{Z}^2 \to \mathbb{C},$$

where and $\phi_k(t) := 2^{-k} \phi(2^{-k} t)$ for some bump function ϕ. In particular, we seek to bound these operators on $\ell^2(\mathbb{Z}^2)$ as efficiently as possible in terms of $|V|$.

Interestingly, the study of these operators leads one to consider an "arithmetic version" of a Kakeya-type problem in the plane, which we approach using a combination of geometric and number-theoretic methods. Motivated by the Furstenberg problem from geometric measure theory, we also consider a discrete directional maximal operator along polynomial orbits,

$$\sup_{v \in V} \left| \sum_{n \in \mathbb{Z}} f(x - v \cdot P(n)) \cdot \phi(n) \right|, \ P \in \mathbb{Z}[-].$$

This is joint work with Laura Cladek.

Noah Kravitz, Yale University
Title: A stronger connection between the Erdős-Burgess and Davenport constants

Abstract: The Erdős-Burgess constant of a semigroup S is the smallest positive integer k such that any sequence over S of length k contains a nonempty subsequence whose elements multiply to an idempotent element of S. In this talk, we focus on the case where S is the multiplicative semigroup of $\mathbb{Z}/n\mathbb{Z}$. The following conjecture of Hao, Wang, and Zhang connects the Erdős-Burgess constant of $\mathbb{Z}/n\mathbb{Z}$ (written $I_r(\mathbb{Z}/n\mathbb{Z})$) and the Davenport constant of $(\mathbb{Z}/n\mathbb{Z})^\times$ (written $\mathsf{D}((\mathbb{Z}/n\mathbb{Z})^\times)$):

$$I_r(\mathbb{Z}/n\mathbb{Z}) = \mathsf{D}((\mathbb{Z}/n\mathbb{Z})^\times) + \Omega(n) - \omega(n),$$

where $\Omega(n)$ denotes the total number of primes in the prime factorization of n (with multiplicity), and $\omega(n)$ denotes the number of distinct primes dividing n. Extending previous results for n squarefree and n a prime power, we prove this conjecture when n is twice a prime power, n has exactly two prime divisors, or n is double the product of two odd prime powers. We also discuss the extension of our techniques to other rings.
Joint work with Ashwin Sah (MIT).

Paolo Leonetti, Graz University of Technology, Austria
Title: Small sets of integers

Abstract: We say that a function $\mu^\star : \mathcal{P}(\mathbf{N}^+) \to [0,1]$ is an *upper quasi-density* if it is subadditive, normalized (i.e., $\mu^\star(\mathbf{N}^+) = 1$), and $\mu^\star(k \cdot X + h) = \frac{1}{k}\mu^\star(X)$ for all $X \subseteq \mathbf{N}^+$ and $k, h \in \mathbf{N}^+$, where $k \cdot X + h := \{kx + h : x \in X\}$. Examples include the upper Buck, upper Pólya, and upper analytic densities, as well as all the upper α-densities (with $\alpha \geq -1$, subsuming the upper logarithmic and upper asymptotic densities). A set $X \subseteq \mathbf{N}^+$ is said to be B-*small* if $\mu^\star(X) = 0$ for every upper quasi-density $\mu^\star$. We establish that a set X is B-small if and only if it belongs to the zero set of the upper Buck density. Then, we show that each of the following sets are B-small: the primes; the perfect powers; the set of all $n \in \mathbf{N}^+$ such that $f(n)$ is prime, where f is a non-constant polynomial with integer coefficients; and the set of integers which can be expressed as sum of two squares.

Huixi Li, University of Nevada, Reno
Title: On two lattice points problems about the parabola

Abstract: In this presentation I will talk about our recent results on the asymptotic formulae with optimal error terms for the number of lattice points under and near a dilation of the standard parabola, the former improving upon a previous result of Popov. These results can be regarded as achieving the square root cancellation in the context of the parabola, whereas its analogues are wide open conjectures for the circle and the hyperbola. Our proofs utilize techniques in Fourier analysis, quadratic Gauss sums and character sums.
Joint work with Jing-Jing Huang.

Neil Lyall, University of Georgia
Title: Hypercubes in sets of positive upper density
Abstract: We plan to discuss the following two results:
1. Any set $A \subseteq \mathbb{R}^{2d}$ of positive upper density necessarily contains a d-dimensional cube of sidelength λ, for all $\lambda \geq \lambda_0(A)$.
2. Any set $A \subseteq \mathbb{Z}^{5d}$ of upper density $\delta > 0$ necessarily contains a d-dimensional cube of sidelength λ, for all $\lambda \geq \lambda_0(A)$ with the property that $\lambda^2 \in q(\delta)\mathbb{Z}$. Results of this type also hold for more general orthotopes and generalize foundational results of Bourgain on non-degenerate simplices in subsets of Euclidean space of positive upper density.
Joint work with Akos Magyar.

Akos Magyar, University of Georgia
Title: Spherical point configurations
Abstract: In the late 80's Bourgain proved that sets of positive upper density of $\mathbb{R}^n$ contain an isometric copy of all sufficiently large dilates of any non-degenerate k-simplex. We show how to extend this result to spherical k-point configurations spanning a $k - 2$-dimensional subspace, both in the model case of finite field geometries as well as in the Euclidean setting, using techniques from additive combinatorics.
Joint work with N. Lyall and H. Parshall.

Ariane Masuda, New York City Tech (CUNY)
Title: Functional graphs of Rédei functions
Abstract: Let $\mathbb{F}_q$ be a finite field of order q and $f \colon \mathbb{F}_q \to \mathbb{F}_q$ be a mapping. The functional graph associated to f is a directed graph where the vertices are labelled by the elements of $\mathbb{F}_q$ and an edge connects a vertex a with a vertex b if and only if $b = f(a)$. In this talk we discuss the structure of the functional graph of a Rédei function $R_n(x, a)$ over $\mathbb{F}_q$ where $n \in \mathbb{N}$ and $a \in \mathbb{F}_q^*$. We show conditions for such graphs to be isomorphic for a fixed q, and present several families of isomorphic graphs.
Joint work with Juliane Capaverde and Virgínia Rodrigues.

Ryan W. Matzke, University of Minnesota - Twin Cities
Title: Maximum size (k, l)-sum-free sets in abelian groups
Abstract: A subset A of a finite abelian group G is called (k, l)-sum-free if the sum of k elements of A never equals the sum of l elements of A. Sets that satisfy this for $k = 2$ and $l = 1$ are often simply called sum-free sets. In 2005, Ben Green and Imre Ruzsa found an explicit formula for the maximum size of a sum-free subset of any finite abelian group. While no generalization of their result exists for finite abelian groups, we shall present a recent result for finite cyclic groups for all $k, l > 0$, and discuss what is currently known.
Joint work with Béla Bajnok.

Steve Miller, Williams College

Title: Recent progress in sumsets, generalized Schreier sets, generalized Zeckendorf decompositions, and prime and square-free Walks

Abstract: This is a report on work in progress from the 2018 SMALL REU and a Spring 2019 research class at Carnegie Mellon. Our first result extends work of Lazarev, Miller and O'Bryant, who showed there was a divot in the probability of a sumset missing exactly k elements (with the set drawn uniformly from all subsets of $\{0, ..., n\}$). We present some theorems and evidence for what happens if instead each element is chosen with a fixed probability p. Our second result concernces Generalized Schreier sets. A Schreier set is a set S of natural numbers with $\min S = |S|$, and the Schreier family containing all Schreier sets is denoted S_1. It has been known that for all n that the sequence of $|\{S \in S_1 | \max S = n\}|$ is the Fibonacci sequence. By defining an α-Schreier set to be a set S of natural numbers with $|\min S/a| = |S|$, we generalize the result and prove that the sequence of $\{S | maxS = n, |\min S/a| = |S|\}$ is of a linear recurrence relation of a higher order, and also consider generalizations with additional constraints. If time permits we will mention some recent results in Generalized Zeckendorf Decompositions and Prime and Square-Free walks (if not we will pose questions on these in the problem session).
Joint work with Tudor Popescu (CMU) and Zimu Xiang (CMU)

Mariusz Mirek, Rutgers University - New Brunswick

Title: Dimension free estimates for the discrete Hardy–Littlewood maximal functions

Abstract: The aim of this talk is to present recent developments in dimension-free estimates in discrete harmonic analysis. This topic has a very natural connection with counting of lattice points in the Euclidean balls and with the Waring problem for the squares. In particular, we show that the discrete Hardy–Littlewood maximal functions associated with the Euclidean balls in $\mathbb{Z}^d$ with dyadic radii have bounds independent of the dimension on $\ell^p(\mathbb{Z}^d)$ for every $p \in [2, \infty]$.
Joint work with J. Bourgain, E.M. Stein, and B. Wróbel.

Akshat Mudgal, University of Bristol

Title: Sums of linear transformations in higher dimensions

Abstract : Given a finite subset A of integers and co-prime natural numbers q and s, we consider the set $q.A + s.A$, that is, the sum of dilates of A. In recent years, finding suitable lower bounds for the cardinality of such sets in terms of $|A|, q$, and s has seen considerable activity. In 2014, Balog and Shakan found sharp estimates for the same, that were tight in both the main term as well as the error term. Subsequently, they considered this problem in higher dimensional integer lattices. In this talk, we present a short survey of these results including our own improvement in the higher dimensional setting.

Rishi Nath, York College (CUNY)
Title: Simultaneous core partitions with nontrivial common divisor

Abstract: The last decade has a tremendous amount of activity around simultaneous (s,t)-core partitions when s and t have no common divisor. Here we relax that condition and explore new bijections and congruences for (s,t)-core and $(\bar{s},\bar{t})$-core partitions when $\gcd(s,t) > 1$.
Joint work with J.B. Gramain and J. A. Sellers.

Mel Nathanson, Lehman College (CUNY)
Title: Matrix scaling and diophantine problems

Abstract: The sequence of matrices obtained from a positive $n \times n$ matrix A by alternate row and column scaling will converge to a doubly stochastic matrix, called the Sinkhorn limit of A. This leads to several problems in diophantine approximation. Moreover, computer experiments suggest that other natural variations of alternate row and column scaling also converge to doubly stochastic matrices that are different from the Sinkhorn limit. The meaning of these limits is unclear.

Timothy Newlin, United States Military Academy, West Point , NY
Title: Twitter response to Munich July 2016 attack: Network analysis of influence

Abstract: Social media platforms in cyberspace provide communication channels for individuals, businesses, as well as state and non-state actors (i.e. individuals and groups) to conduct messaging campaigns. This research analyzes the spheres of influence that arose around the keyword *#Munich* on Twitter following an active shooter event at a Munich shopping mall in July 2016. Tweets utilizing *#Munich* were captured beginning one hour after the shooting was reported and data collection ended approximately one month later. We constructed a cumulative network and several daily networks from this data, and analyzed community evolution using the Louvain algorithm. The analysis of how such communities change over time has many applications to both encourage and discourage the effectiveness of an information messaging campaign. The largest communities observed in the early portions of the data disappear from the *#Munich* conversation within seven days. The politically charged nature of many of these communities suggests their activity migrated to other hashtags (i.e. conversation topics). Future analysis of Twitter activity might focus on tracking communities across topics and time. Finally, we suggest techniques on methods for constructing synthetic networks to further understand how these communities of influence adapt over time.
Joint work with Ivan Bermudez, Jan-Daniel Cleven, Ralucca Gera,Erik Kiser.

Kevin O'Bryant, College of Staten Island (CUNY)
Title: Explicit bounds on the number of zeros of Dirichlet L-functions

Abstract: We will present new explicit results on the number of zeros of Dirichlet L-series with height at most T.

Andrew Odesky, University of Michigan
Title: Characterizing polynomial sequences in p-adic interpolation
Abstract: Consider a sequence $s : \mathbb{N} \to k$ valued in a number field k. Given a place ν of k lying over a prime p, one may ask under what circumstances the sequence s is interpolated by a nice function $f : \mathbb{Z}_p \to k_\nu$, where k_ν is the completion of k for the place ν. For instance, if s is given by the values of some polynomial, then it may be interpolated by ν-adic analytic functions for any place ν of k. The converse to this example presents an interesting problem in p-adic analysis: How to characterize the polynomial sequences among all sequences in terms of natural p-adic conditions?

An elegant theorem of R.R. Hall from 1971 gives one satisfactory answer: If s preserves congruences and its generating function has sufficiently large Archimedean radius of convergence, then it is polynomial. Work of D.L. Hilliker and E.G. Straus gives another answer when s may be interpolated by analytic functions. A theorem of B. Dwork from his famous paper on the rationality of the zeta function of an algebraic variety gives another converse result.

In this talk I will present new converse results of a general type which subsume the results mentioned above.

Jun Seok Oh, University of Graz
Title: An inverse zero-sum problem for some non-abelian groups
Abstract: Let G be a finite group. By a sequence over G, we mean a finite unordered sequence of terms from G, with repetition allowed. We say that it is a product-one sequence if its terms can be ordered such that their product equals the identity element of G. The large Davenport constant $\mathsf{D}(G)$ is the maximal length of a minimal product-one sequence, that is a product-one sequence which cannot be factored into two non-trivial product-one subsequences.

The study of the Davenport constant of finite abelian groups is a central topic in zero-sum theory since the 1960s. Both the direct problem, asking for the precise value of the Davenport constant in terms of group invariants, as well as the associated inverse problem, asking for the structure of extremal sequences, have received wide attention in the literature. Even in the abelian case, little is known beyond groups of rank at most two.

In recent years, Geroldinger and Grynkiewicz derived the precise value the large Davenport constant for groups having a cyclic index 2 subgroup, and dihedral and dicyclic groups belong to this class of groups. In this talk, we provide explicit characterizations of all minimal product-one sequences of length $\mathsf{D}(G)$ over dihedral and dicyclic groups.
Joint work with Qinghai Zhong.

Yumeng Ou, Baruch College (CUNY)
Title: Applications of decoupling in distance set problems
Abstract: The Falconer distance set conjecture says that if a compact set in $\mathbb{R}^d$ has Hausdorff dimension greater than $\frac{d}{2}$, then its distance set must have positive Lebesgue measure. The conjecture remains open in all dimensions $d \geq 2$. There have been many new developments recently achieved towards it as well as some

related distance set problems, and the celebrated Bourgain-Demeter decoupling inequality played an important role. We will talk about some of these connections.

Nikita Pereverzin, United States Military Academy, West Point , NY
Title: The God's Number for the n x n Rubik's Slide
Abstract: God's Number represents the maximum of the minimum numbers of transformations from any initial configuration to any possible final configuration of a n x n Rubik's Slide, if the game is played with no error. The different moves (shifting and rotating) form a group of permutations. Jones et al. leveraged this group structure to develop different approaches varying in complexity to derive the God's Number. Jones et al. proved that the God's Number is the diameter of the resulting Cayley Graph and also found a way to derive the God's Number from coset decompositions. Using these decompositions, Jones et al. was able to calculate the God's Number for smaller sized Rubik's Slides.

The objective of this project is to advance the work of Jones et al. to find the God's Number for the n x n Rubik's Slide. To this end, we built a scala program that generates the adjacency matrix for any Rubik's Slide and then calculates the spectral radius of the matrix. Additionally, we developed a machine learning algorithm to simulate the Rubik's Slide game for $n = 2$ and $n = 3$ and statistically estimate the God's Number.

Cosmin Pohoata, California Institute of Technology
Title: Sets without 4APs but with many 3APs
Abstract: It is a classical theorem of Roth that every dense subset of $\{1, \ldots, N\}$ contains a nontrivial three-term arithmetic progression. Quantitatively, results of Sanders, Bloom, and Bloom-Sisask tell us that subsets of relative density at least $1/(\log N)^{1-\epsilon}$ already have this property. In this talk, we will discuss the existence of sets of N integers that, unlike $\{1, \ldots, N\}$, do not contain nontrivial four-term arithmetic progressions, but still have the property that all subsets of relative density at least $1/(\log N)^{1-\epsilon}$ must contain a three-term arithmetic progression. We shall also discuss the analogous story over $\mathbb{F}_q^n$, where there exist four-term progression free sets A such that, for every subset $A' \subset A$ with $|A'| \geq |A|^{1-c}$, the set A' contains a three term arithmetic progression.
Joint work with Oliver Roche-Newton.

Yuneid Puig de Dios, University of California, Riverside
Title: Kříž's theorem via dynamics of linear operators
Abstract: The existence of a set $A \subseteq \mathbf{Z}$ of positive upper Banach density such that $A - A := \{m - n : m, n \in A, m > n\}$ does not contain a set of the form $E - E$ with E piecewise syndetic is in essence the content of a popular result due to Kříž in 1987, in which he used a graph-theoretical approach. More recently, other proofs of this result have been given using combinatorial number theory. Our goal here is to show that a stronger result than the one given by Kříž can be obtained, and that this can be done via operator theory, namely using dynamics of linear operators on Banach spaces.

Alex Rice, Millsaps College
Title: Deligne polynomials in difference sets
Abstract: We will discuss strong upper bounds on the density of sets of integers lacking differences of the form $h(m,n)$, where $h \in \mathbb{Z}[x,y]$ satisfies appropriate smoothness conditions inspired by finite field exponential sum estimates of Deligne. Joint work with John Doyle.

Misha Rudnev, University of Bristol
Title: On growth in groups $SL_2(\mathbb{F}_p)$ and $\mathrm{Aff}(\mathbb{F}_p)$
Abstract: The talk will focus on the geometric basics that enable one to prove reasonable quantitative estimates for the rate of growth, by group multiplication, in the above groups. The first qualitative results of this kind are due to Helfgott. They are a fountainhead of today's rich theory of finite approximate groups and its applications, where several papers of Bourgain and collaborators played a pivotal role. Apart from attempting to elucidate the structure of the proofs, the talk will address the relationship between growth in non-commutative groups and the sum-product phenomenon.

Alisa Sedunova, MPIM Bonn
Title: Intersections of binary quadratic forms in primes and the paucity phenomenon
Abstract: Let $r(n)$ be the function that counts the number of ways to represent a natural number n as a sum of two positive squares. The number of solutions to $a^2 + b^2 = c^2 + d^2 \leq x$, hence, the second moment of $r(n)$, in integers is a well-known result, while if one restricts all the variables to primes Erdős showed that only the diagonal solutions, namely, the ones with $a = c$, $b = d$ contribute to the main term, hence there is a pacuity of the off-diagonal solutions. In this talk we discuss the work in progress about second moments of $r(n)$, when *some* of the variables are restricted to primes and discuss the existence of paucity for the off-diagonal solutions in such problems. The methods are largely based on Hooley technique to tackle (on GRH) the Hardy-Littlewood problem about the representations $N = p + a^2 + b^2$, where p is a prime and a, b are integers, some related works of Plaksin (based on the unconditional resolution of the Hardy-Littlewood problem by Linnik) and more recent results of S. Daniel.

Steven Senger, Missouri State University
Title: Chains of points determined by distances
Abstract: We consider the following generalization of the celebrated unit distance problem: Given any large finite set of n points in the plane, and k distances, $d_1, d_2, \ldots, d_k$, how many $(k+1)$-tuples of points can there be where subsequent pairs of points are separated by the indicated distances. Such $(k+1)$-tuples are called chains. We present new upper and lower bounds for the maximum possible number of chains. We also present related results in other settings: three dimensions, finite fields, finite rings, and an analogous question about dot products.

Adam Sheffer, Baruch College (CUNY)
Title: Lower bounds for incidences with hypersurfaces
Abstract: Geometric incidences are a family of combinatorial problems that have a large variety of applications. Bourgain often relied on incidences, using them to obtain results in Number Theory (e.g., energy of Gaussian numbers), Theoretical Computer Science (2-source extractors), and Harmonic Analysis (restriction problems). Bourgain and Demeter used incidences to study a discrete Fourier restriction to the four- and five-dimensional sphere. We will show how reversing the direction of their proof leads to new lower bounds for a family of incidence problems. This leads to one of few cases where an incidence problem is solved up to sub-polynomial factors.

Ilya Shkredov, Steklov Mathematical Institute, Moscow, Russia
Title: SL_2–actions, modular hyperbolas, and Kloosterman sums
Abstract: We are going to talk about the connection between growth in $SL_2(\mathbf{F})$ for various fields $\mathbf{F}$ and a series of problems of Number Theory and Additive Combinatorics such as Zaremba's conjecture form the theory of continued fractions, distribution of points onto modular hyperbolas, and estimates of bilinear forms of Kloosterman sums.

Satyanand Singh, New York City Tech (CUNY)
Title: Generating terms of Nathanson's lambda sequences
Abstract: We consider the set

$$A_n = \bigcup_{j=0}^{\infty} \{\varepsilon_j(n) \cdot n^j : \varepsilon_j(n) \in \{0, \pm 1, \pm 2, ..., \pm \lfloor n/2 \rfloor\}\}.$$

Let $\mathcal{S}_\mathcal{A} = \bigcup_{a \in \mathcal{A}} A_a$, where $\mathcal{A} \subseteq \mathbb{N}$. We denote by $\lambda_\mathcal{A}(h)$ the smallest positive integer that can be represented as a sum of h, and no less than h elements in $\mathcal{S}_\mathcal{A}$. Nathanson studied the properties of the $\lambda_\mathcal{A}(h)$ sequence and posed the problem of finding the values of $\lambda_\mathcal{A}(h)$, which are very important in geometric group theory and additive number theory. When $\mathcal{A} = \{2, j\}$, we represent $\lambda_\mathcal{A}(h)$ by $\lambda_{2,j}(h)$. For fixed $h \in \{1, 2, 3\}$, the values of $\lambda_{2,j}(h)$ are known as j runs over the odd integers bigger than 1. In this presentation, we extend this result further by illustrating how to generate $\lambda_{2,j}(4)$ as j runs over the odd integers bigger than 1. In particular we will show that $\lambda_{2,3}(4) = 150$ and $\lambda_{2,5}(4) = 83$. Our technique involve establishing the insolubility of certain exponential diophantine equations.

Jozsef Solymosi, University of British Columbia, Canada
Title: Problems and results on the sum-product problem of Erdős and Szemerédi
Abstract: I will talk about the sum-product problem.

Jonathan Sondow, New York

Title: On Carmichael and polygonal numbers, Bernoulli polynomials, and sums of base-p digits

Abstract: We give a new characterization of the set C of Carmichael numbers in the context of p-adic theory, independently of the classical results of Korselt and Carmichael. The characterization originates from a surprising link to the denominators of the Bernoulli polynomials via the sum-of-base-p-digits function. More precisely, we show that such a denominator obeys a triple-product identity, where one factor is connected with a p-adically defined subset S of the squarefree integers that contains C. This leads to the definition of a new subset C' of C, called the primary Carmichael numbers. Subsequently, we establish that every Carmichael number equals an explicitly determined polygonal number of a special type. Finally, the set S is covered by modular subsets S_d (for $d \geq 1$) that are related to the Knoedel numbers, where $C = S_1$ is a special case.
Joint work with Bernd C. Kellner. See arxiv.org/abs/1902.10672 for our preprint.

Wenbo Sun, Ohio State University

Title: Sarnak's Conjecture for nilsequences on arbitrary number fields, and applications

Abstract: Sarnak's conjecture is a hot topic in number theory and ergodic theory in recent years. The conjecture says that the Möbius function is orthogonal to any sequence coming from a dynamical system of low complexity. Nowadays, many special cases of the Sarnak's conjecture have been proven and many applications have been found. In this lecture, we shall talk about generalizations of Sarnak's conjecture to the ring of integers of an arbitrary number field, and present new results in this direction. We shall also talk about its application in combinatorics. For example, we show that for any finite coloring of the Gaussian integers $\mathbf{Z}[i]$, there exist x and y distinct and non-zero of the same color such that $x^2 - y^2 = n^2$ for some Gaussian integer n.

Van Vu, Yale University

Title: On square sum-free sets

Abstract: How many numbers can one choose from $\{1, 2, ...n\}$ so that no subset sum is a square? This old problem of Erdős has a fascinating history, linking many leading researchers of the field of additive combinatorics (some in a way you would not expect).

Aled Walker, Trinity College, Cambridge

Title: Linear inequalities in primes

Abstract: For over 80 years it has been known how to use Fourier analysis to prove an asymptotic formula for the number of three-term arithmetic progressions of primes less than N. These methods also apply to m simultaneous linear equations in primes, provided that the number of prime variables is at least $2m + 1$. Much more recently, by the revolutionary work of Green-Tao-Ziegler, it was shown how to count the number of solutions to such systems provided that there were at least $m + 2$ prime variables. In this talk we will discuss the related setting of linear

inequalities in primes. By adapting the methods of Green and Tao, together with some additional ideas, we will prove an asymptotic formula for the number of prime solutions to systems of m simultaneous inequalities with at least $m + 2$ variables. This greatly improves upon the existing analytic methods, which, by analogy with linear equations, require at least $2m + 1$ variables.

Hong Wang, MIT
Title: Incidences between points and tubes
Abstract: We prove analogues of Szemerédi-Trotter theorem and other incidence theorems for δ–approximate incidences, assuming that the δ–tubes are well-spaced in a strong sense.
Joint work with Larry Guth and Noam Solomon.