CANT 2020: Zoom Conference

Eighteenth Annual Workshop on
Combinatorial and Additive Number Theory
CUNY Graduate Center (via Zoom)
June 1 - 5, 2020

Abstracts

(1) **Sukumar Das Adhikari**, Ramakrishna Mission Vivekananda Educational and Research Institute (RKMVERI), India
Title: Weighted generalization of a theorem of Gao
Abstract: Gao proved that for a finite abelian group of order n, we have $E(G) = D(G) + n - 1$, where $D(G)$ is the Davenport constant of G and $E(G)$ is defined to be the smallest natural number k such that any sequence of k elements in G has a subsequence of length n whose sum is zero. We shall discuss a weighted generalization of the above relation of Gao.

(2) **Theresa C. Anderson**, Brown University
Title: How numbers interact with curves
Abstract: We show how discrete versions of averaging operators from harmonic analysis behave drastically differently from their continuous counterparts. We do this through examples: starting with a bit of history and ending by sampling recent results. We plan to discuss the case of the spherical maximal function, introducing several variants, such as averaging along primes, which allow us to describe precise lattice point distribution.

(3) **George E. Andrews**, Pennsylvania State University
Title: Separable integer partition (SIP) classes
Abstract: Three of the most classical and well-known identities in the theory of partitions concern: (1) the generating function for $p(n)$ (Euler); (2) the generating function for partitions into distinct parts (Euler), and (3) the generating function for partitions in which parts differ by at least 2 (Rogers-Ramanujan). The lovely, simple argument used to produce the relevant generating functions is mostly never seen again. Actually, there is a very general theorem here which we shall present. We then apply it to prove two familiar theorems; (1) Göllnitz-Gordon, and (2) Schur 1926. We also consider an example where the series representation for the partitions in question is new. We close with an application to "partitions with n copies of n."

(4) **Michael Bennett**, University of British Columbia
Title: Differences between perfect powers
Abstract : In this talk, I will survey a variety of arithmetic problems related to the sequence of differences between perfect powers, highlighting what is known, what is expected to be true, and what is (possibly) within range of current technology. I shall discuss some recent joint work with Samir

Siksek on a number of related classical polynomial-exponential equations.

(5) **Gautami Bhowmik**, Université de Lille, France
Title: Asymptotics of products of L-functions
Abstract: Among the analytic properties of L-functions, we are interested in their mean values, called moments. We will mention classical results, and then treat mixed moments of the product of Hecke L-functions and symmetric square L-functions associated to primitive cusp forms.
Joint work with O. Balkanova, D. Frolenkov and N. Raulf.

(6) **Arie Bialostocki**, University of Idaho
Title: Zero-Sum Ramsey theory: Origins, present, and future
Abstract: As for the origins, I will describe the birth of the Erdős-Ginzburg-Ziv theorem as I learned it from the late A. Ziv and A. Ginzburg in 2003. A stimulating conversation with V. Milman around 1980 led me to a broader view of Ramsey Theory. I shared some of the ideas with my friends Y. Caro and Y. Roditty. Y. Caro took a slightly different turn and made several significant contributions. In the mid 80s I started my 15-year collaboration with my colleague P. Dierker. In 1989, R. Graham learned about the zero-sum tree conjecture and popularized it. It was solved by Z. Füredi and D. Kleitman, and, independently, by A. Schrijver and P. D. Seymour. In 1990 I visited Australia and was introduced to a young student M. Kisin, who made a significant contribution toward the multiplicity conjecture, solved asymptotically by Z. Füredi and D. Kleitman. Another milestone was my joint paper with P. Erdős and H. Lefman, which was the beginning of zero-sum theory on the integers. A few of my Ph.D students and some of my REU students, among them D. Grynkiewicz, made some significant contributions. But I believe that my last Ph.D student, T.D. Luong, paved the way to future research, which I will call vanishing polynomials. Though the abstract describes mainly the history, much of the lecture will be devoted to the present and the future.

(7) **Pierre-Yves Bienvenu**, Université de Lyon
Title: Additive bases in infinite abelian semigroups, I
An additive basis A of a semigroup T is a subset such that every element of T, up to a finite set of exceptions, may be written as a sum of one and the same number h of elements from the basis. The minimal such number h is called the order of the basis. We study bases in a class of infinite abelian semigroups, which we term translatable semigroups. These include all infinite abelian groups as well as the semigroup of nonnegative integers. We analyze the "robustness" of bases. Such discussions have a long history in the semigroup $\mathbf{N}$, originating in the work of Erdős and Graham, continued by Deschamps and Farhi, Nathanson and Nash, Hegarty.... Thus we consider essential subsets of a basis A, that is, finite sets F such that $A \setminus F$ is no longer a basis, and which are minimal. We show that any basis has only finitely many essential subsets, and we bound the number of essential subsets of cardinality k of a basis of order h in terms of h and k.

Joint work with Benjamin Girard and Thai Hoang Lê.

(8) **Arindam Biswas**, Technion - Israel Institute of Technology
Title: On minimal complements and co-minimal pairs in groups
Abstract: Given two non-empty subsets $W, W' \subseteq G$ in a group G, the set W' is said to be a complement to W if $W \cdot W' = G$ and it is minimal if no proper subset of W' is a complement to W. The notion was introduced by Nathanson in the course of his study of natural arithmetic analogues of the metric concept of nets in the setting of the integers. A notion stronger than minimal complements is that of a co-minimal pair. A pair of subsets (W, W') is a co-minimal pair if $W \cdot W' = G$ and W is minimal with respect to W' and vice-versa. In this talk we shall mainly concentrate on abelian groups and show some recent developments on the existence and the non-existence of minimal complements and of co-minimal pairs.
Joint work with Jyoti Prakash Saha.

(9) **Yong-Gao Chen**, Nanjing Normal University, P. R. China
Title: On a problem of Erdős, Nathanson and Sárközy
Abstract: In 1988, Erdős, Nathanson and Sárközy proved that if A is a set of nonnegative integers with lower asymptotic density $1/k$, where k is a positive integer, then $(k+1)A$ must contain an infinite arithmetic progression with difference at most $k^2 - k$, where $(k+1)A$ is the set of all sums of $k+1$ elements of A. They asked if $(k+1)A$ must contain an infinite arithmetic progression with difference at most $O(k)$. In this talk, we answer this problem negatively by proving that, for every sufficiently large integer k, there exists a set A of nonnegative integers with the lower asymptotic density $1/k$ such that $(k+1)A$ does not contain an infinite arithmetic progression with difference less than $k^{1.5}$.
Joint work with Ya-Li Li.

(10) **Alex Cohen**, Yale University
Title: A Sylvester-Gallai result in the complex plane
Abstract: We show that for a Sylvester-Gallai configuration in $\mathbb{C}^2$ lying on a family of m concurrent lines, each line in the family can contain at most $3m - 9$ points of the set, not including the common point. This implies that many points lying on a family of concurrent lines must admit an ordinary line. We also introduce a conjecture which would improve this bound to $m - 1$, which is sharp. Our approach involves ordering points by their real part, which is a new technique for studying complex line arrangements.

(11) **Gabriel Conant**, University of Cambridge, UK
Title: Small tripling with forbidden bipartite configurations
Abstract: A finite subset A of a group G is said to have *k-tripling* if $|AAA| \leq k|A|$. I will report on recent joint work with A. Pillay, in which we study the structure finite sets A with k-tripling, under the additional assumption that the bipartite graph relation $xy \in A$ omits some induced subgraph of a fixed size d. In this case, we show that A is approximately a union of a bounded number of translates of a coset nilprogression in G

of bounded rank and step (where "bounded" is in terms of k, d, and a chosen approximation error $\epsilon > 0$). Our methods combine the work of Breuillard, Green, and Tao on the structure of approximate groups, together with model-theoretic tools based on the study of groups definable in NIP theories.

(12) **Michael Curran**, Williams College
Title: Ehrhart theory and an explicit version of Khovanskii's theorem
Abstract: A remarkable theorem due to Khovanskii asserts that for any finite subset A of an abelian group, the cardinality of the h-fold sumset hA grows like a polynomial for all sufficiently large h. However, neither the polynomial nor what sufficiently large means are understood in general. We use Ehrhart theory to give a new proof of Khovanskii's theorem when $A \subset \mathbb{Z}^d$ that gives new insights into the growth of the cardinality of sumsets. Our approach allows us to obtain explicit formulae for $|hA|$ whenever $A \subset \mathbb{Z}^d$ contains $d+2$ points that are valid for *all* h. In the case that the convex hull Δ_A of A is a d-dimensional simplex, we can also show that $|hA|$ grows polynomially whenever $h \geq \mathrm{vol}(\Delta_A) \cdot d! - |A| + 2$.

(13) **Robert W. Donley, Jr.**, Queensborough Community College (CUNY)
Title: Semi-magic matrices for dihedral groups
Abstract: If the finite group G acts on a finite set X, then G may be represented by a subgroup of permutation matrices, which in turn generate an algebra of semi-magic matrices. Recall that a semi-magic matrix is a square matrix with complex coefficients whose rows and columns have a common line sum. In the case of dihedral groups, we apply character theory to recover the known counting formula for semi-magic matrices with fixed line sum and coefficients in the natural numbers.

(14) **Shalom Eliahou**, Université du Littoral Côte d'Opale, France
Title: Some recent results on Wilf's conjecture
Abstract: A *numerical semigroup* is a submonoid S of the nonnegative integers with finite complement. Its *conductor* is the smallest integer $c \geq 0$ such that S contains all integers $z \geq c$, and its *left part* L is the set of all $s \in S$ such that $s < c$. In 1978, Wilf asked whether the inequality $n|L| \geq c$ always holds, where n is the least number of generators of S. This is now known as Wilf's conjecture. In this talk, we present some recent results towards it, using tools from commutative algebra and graph theory.

(15) **Christian Elsholtz**, Graz University of Technology, Austria
Title: Sums of unit fractions
Abstract: Let $f_k(m,n)$ denote the number of solutions of $\frac{m}{n} = \frac{1}{x_1} + \cdots + \frac{1}{x_k}$ in positive integers x_i. The case $k = 2$ is essentially a question on a divisor function, and the case $k = 3$ is closely related to a sum of certain divisor functions. For the case $k = 3, m = 4$ Erdős and Straus conjectured that

$$f_3(4,n) > 0 \text{ for all } n > 1.$$

The case $m = n = 1$ received special attention, and even has applications in discrete geometry. We give a survey on previous results and report on new results over the last years.

Theorem 1: There are infinitely many primes p with

$$f_3(m,p) \gg \exp(c_m \frac{\log p}{\log\log p}).$$

Theorem 2: For fixed m and almost all integers n one has:

$$f_3(m,n) \gg (\log n)^{\log 3 + o_m(1)}.$$

Theorem 3: $f_3(4,n) = O_\varepsilon\left(n^{3/5+\varepsilon}\right)$, for all $\varepsilon > 0$.
There are related but more complicated bounds when $k \geq 4$.

Joint work with T. Browning, S. Planitzer, and T. Tao.

(16) **Leonid Fel**, Technion – Israel Institute of Technology, Israel
Title: A sum of negative degrees of the gaps values in two-generated numerical semigroups and identities for the Hurwitz zeta function
Abstract: We derive an explicit expression for an inverse power series over the gaps values of numerical semigroups generated by two integers. It implies a set of identities for the Hurwitz zeta function $\zeta(n,q)$ including the multiplication theorem for $\zeta(n,q)$.

Joint work with Takao Komatsu and Ade Irma Suriajaya.

(17) **Michael Filaseta**, University of South Carolina
Title: Two excursions in digitally delicate primes
Abstract: In 1978, Murray Klamkin asked whether there are prime numbers such that if any digit in the prime is replaced by any other digit, the resulting number is composite. In 1979, several examples were published together with a proof by Paul Erdős that infinitely many such primes exist. Following the terminology of Jackson Hopper and Paul Pollack, we call such primes “digitally delicate.” The smallest digitally delicate prime is 294001. In this talk, we discuss some of the history surrounding digitally delicate primes, implications of prior work, and recent work by the speaker with Jeremiah Southwick and Jacob Juillerat.

(18) **Alfred Geroldinger**, University of Graz, Austria
Title: Zero-sum sequences over finite abelian groups and their sets of lengths
Abstract: Let G be an additively written abelian group. A (finite unordered) sequence $S = g_1 \ldots g_\ell$ of terms from G (with repetition allowed) is said to be a *zero-sum sequence* if $g_1 + \ldots + g_\ell = 0$. Every zero-sum sequence S can be factored into minimal zero-sum sequences, say $S = S_1 \ldots S_k$. Then k is called a factorization length of S and $\mathsf{L}(S) \subset \mathbb{N}$ denotes the set of all factorization lengths of S. We consider the system $\mathcal{L}(G) = \big\{\mathsf{L}(S) \colon S \text{ is a zero-sum sequence over } G\big\}$ of all sets of lengths.

Let G and G' be finite abelian groups. A standing conjecture states that $\mathcal{L}(G) = \mathcal{L}(G')$ if and only if G and G' are isomorphic (provided that $|G| \geq 4$). If $\mathcal{L}(G) = \mathcal{L}(G')$, then $\mathsf{D}(G) = \mathsf{D}(G')$, where $\mathsf{D}(G)$ and $\mathsf{D}(G')$ are the Davenport constants of G and of G'. We show that each of the systems, $\mathcal{L}(C_m)$ and $\mathcal{L}(C_2^{m-1})$ with $m \geq 7$, is incomparable (with respect to set theoretical inclusion) with any other system $\mathcal{L}(G)$ having the same Davenport constant. This is joint work with Wolfgang A. Schmid (see https://arxiv.org/abs/2005.03316).

(19) **Kåre Gjaldbæk**, CUNY Graduate Center
Title: Noninjectivity of nonzero discriminant polynomials and applications to packing polynomials
Abstract : We show that an integer-valued quadratic polynomial on $\mathbb{R}^2$ can not be injective on the integer lattice points of any subset of $\mathbb{R}^2$ containing an affine convex cone if its discriminant is nonzero. A consequence is the non-existence of quadratic packing polynomials on irrational sectors of $\mathbb{R}^2$. The result also simplifies a classical proof of the Fueter-Pólya Theorem, which states that the two Cantor polynomials are the only quadratic polynomials bijectively mapping $\mathbb{N}_0^2$ onto $\mathbb{N}_0$.

(20) **Daniel Glasscock**, University of Massachusetts, Lowell
Title: Uniformity in the dimension of sumsets of p- and q-invariant sets, with applications in the integers
Abstract: Harry Furstenberg made a number of conjectures in the 60's and 70s seeking to make precise the heuristic that there is no common structure between digit expansions of real numbers in different bases. Recent solutions to his conjectures concerning the dimension of sumsets and intersections of times p- and q-invariant sets now shed new light on old problems. In this talk, I will explain how to use tools from fractal geometry and uniform distribution to get uniform estimates on the Hausdorff dimension of sumsets of times p- and q-invariant sets. I will explain how these uniform estimates lead to applications in the integers: the dimension of a sumset of a p-invariant set and a q-invariant set in the integers is as large as it can be.

This talk is based on joint work with Joel Moreira and Florian Richter.

(21) **Lajos Hajdu**, University of Debrecen, Hungary
Title: The validity of Skolem's conjecture for a family of exponential equations
Abstract: According to Skolem's conjecture, if an exponential Diophantine equation is not solvable, then it is not solvable modulo an appropriately chosen modulus. Besides several concrete equations, the conjecture has only been proved for rather special cases. In the talk we present a new theorem proving the conjecture for equations of the form $x^n - by_1^{k_1} \dots y_\ell^{k_\ell} = \pm 1$, where $b, x, y_1, \dots, y_\ell$ are fixed integers and $n, k_1, \dots, k_\ell$ are non-negative integral unknowns. Note that the family includes the famous equations $x^n - y^k = 1$ and $\frac{x^n-1}{x-1} = y^k$ with x, y fixed.

Joint with A. Bérczes and R. Tijdeman.

(22) **Norbert Hegyvárí**, Eötvös University and Rényi Institute, Budapest
Title: Hilbert cubes meet arithmetic sets
Abstract: In 1978, Nathanson obtained several results on sumsets contained

in infinite sets of integers. Later the author investigated how big a Hilbert cube avoiding a given *infinite* sequence of integers can be.

In the present talk, we show that an additive Hilbert cube, in *prime fields* of sufficiently large dimension, always meets certain kinds of arithmetic sets, namely, product sets and reciprocal sets of sumsets satisfying certain technical conditions.

Joint work with Peter P. Pach.

(23) **Harald Helfgott**, Universität Göttigen/CNRS
Title: Optimality of the logarithmic upper-bound sieve, with explicit estimates
Abstract: At the simplest level, an upper bound sieve of Selberg type is a choice of $\rho(d)$, $d \leq D$, with $\rho(1) = 1$, such that

$$S = \sum_{n \leq N} \left(\sum_{d|n} \mu(d)\rho(d) \right)^2$$

is as small as possible. The optimal choice of $\rho(d)$ for given D was found by Selberg. However, for several applications, it is better to work with functions $\rho(d)$ that are scalings of a given continuous or monotonic function η. The question is then: What is the best function η, and how does S for given η and D compare to S for Selberg's choice?

The most common choice of η is that of Barban-Vehov (1968), which gives an S with the same main term as Selberg's S. We show that Barban and Vehov's choice is optimal among all η, not just (as we knew) when it comes to the main term, but even when it comes to the second-order term, which is negative and which we determine explicitly.

Joint work with Emanuel Carneiro, Andrés Chirre and Julian Mejía-Cordero.

(24) **Neil Hindman**, Howard University
Title: Tensor products in $\beta(\mathbb{N} \times \mathbb{N})$
Abstract: Given a discrete space S, the Stone-Čech compactification βS of S consists of all of the ultrafilters on S. If $p \in \beta S$ and $q \in \beta T$, then the *tensor product*, $p \otimes q \in \beta(S \times T)$ is defined by

$$p \otimes q = \{A \subseteq S \times T : \{x \in S : \{y \in T : (x, y) \in A\} \in q\} \in p\}.$$

Tensor products of members of $\beta\mathbb{N}$ are intimately related to addition on $\mathbb{N}$. If $\sigma : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is defined by $\sigma(s, t) = s + t$ and $\widetilde{\sigma} : \beta(\mathbb{N} \times \mathbb{N}) \to \beta\mathbb{N}$ is its continuous extension, then for any $p, q \in \beta\mathbb{N}$, $\widetilde{\sigma}(p \otimes q) = p + q$. Among our results are the facts that if $S = (\mathbb{N}, +)$ or $S = (\mathbb{R}_d, +)$, where $\mathbb{R}_d$ is $\mathbb{R}$ with the discrete topology, and $S^* = \beta S \setminus S$, then $S^* \otimes S^*$ misses the closure of the smallest ideal of $\beta(S \times S)$ and $\beta S \otimes \beta S$ is not a Borel subset of $\beta(S \times S)$.

Joint work with Dona Strauss.

(25) **Brian Hopkins**, Saint Peter's University
Title: Restricted multicompositions

Abstract: In 2007, George Andrews introduced k-compositions, a generalization of integer compositions, where each summand has k possible colors, except for the final part which must be color 1. Last year, Stéphane Ouvry and Alexios Polychronakos introduced g-compositions which allow for up to $g-2$ zeros between parts. Although these do not have the same definition and came from very different motivations (number theory and quantum mechanics, respectively), we will see that they are equivalent. One reason these are compelling combinatorial objects is their count: there are $(k+1)^{n-1}$ k-compositions of n. Results from standard integer compositions can have interesting generalizations. For example, there are three types of restricted compositions counted by Fibonacci numbers—parts 1 & 2, odd parts, and parts greater than 1. We will explore the diverging families of recurrences that arise from applying these restrictions to multicompositions.

(26) **Robert Hough**, SUNY at Stony Brook
Title: The 15 puzzle problem
Abstract: An n^2-1 puzzle is a children's toy with n^2-1 numbered pieces on an $n \times n$ grid, with one missing piece. A move in the puzzle consists of sliding an adjacent numbered piece into the location of the missing piece. I will discuss joint work with Yang Chu which studies the asymptotic mixing of an n^2-1 puzzle when random moves are made. The techniques involve characteristic function methods for studying the renewal process described by the sequence of moves of one or several pieces.

(27) **Jing-Jing Huang**, University of Nevada, Reno
Title: Diophantine approximation on affine subspaces
Abstract: We extend the classical theorem of Khintchine on metric diophantine approximation to affine subspaces of $\mathbf{R}^n$. In order to achieve this it is necessary to impose some condition on the diophantine exponent of the matrix defining the affine subspace. Our result actually concerns the more general Hausdorff measure, which is known as the generalized Baker-Schmidt problem. We solve this problem by establishing optimal estimates for the number of rational points lying close to the affine subspace.

(28) **Alex Iosevich**, University of Rochester
Title: An approximate Szemeŕedi-Trotter incidence theorem
Abstract: We are going to discuss the Szemerédi-Trotter incidence theorem for points and annuli, as well as some related problems in analysis and number theory.

(29) **Brad Isaacson**, New York City Tech (CUNY)
Title: Formulas for some exponential and trigonometric character sums
Abstract: We express three different, yet related, character sums by generalized Bernoulli numbers. Two of these sums are generalizations of sums introduced and studied by Berndt and Arakawa-Ibukiyama-Kaneko in the context of the theory of modular forms. A third sum generalizes a sum already studied by Ramanujan in the context of theta function identities.

Our methods are elementary, relying on basic facts from algebra and number theory.

(30) **Renling Jin**, College of Charleston
Title: Szemerédi's theorem, nonstandardized and simplified
Abstract: We will present a "simple" nonstandard proof of Szemerédi's theorem for four-term arithmetic progressions based on Terence Tao's interpretation of Szemerédi's original idea.

(31) **William Keith**, Michigan Technological University
Title: Part-frequency matrices of partitions: New developments and related bijections
Abstract: As one of those mathematical confluences that sometimes happen, in recent years several researchers appear to have independently developed the same generalization of Glaisher's bijection on partitions: a natural matrix construction with wide application in combinatorial proofs. In this talk we shall illustrate the core idea, give some new theorems employing it, and suggest some questions that might be of interest for further exploration.

(32) **Dylan King**, Wake Forest University
Title: Distribution of missing sums in correlated sumsets
Abstract: Given a finite set of integers A, its sumset is $A+A := \{a_i + a_j \mid a_i, a_j \in A\}$. We examine $|A+A|$ as a random variable, where $A \subset I_n = [0, n-1]$, the set of integers from 0 to $n-1$, so that each element of I_n is in A with a fixed probability $p \in (0,1)$. Martin and O'Bryant studied the case in which $p = 1/2$ and found a closed form for $\mathbb{E}[|A+A|]$. Lazarev, Miller, and O'Bryant extended the result to find a numerical estimate for $\mathrm{Var}(|A+A|)$ and bounds on $m_{n\,;\,p}(k) := \mathbb{P}(2n-1-|A+A| = k)$. Their primary tool was a graph theoretic framework which we now generalize to provide a closed form for $\mathbb{E}[|A+A|]$ and $\mathrm{Var}(|A+A|)$ for all $p \in (0,1)$ and establish good bounds for $\mathbb{E}[|A+A|]$ and $m_{n\,;\,p}(k)$. We extend the graph theoretic framework originally introduced by Lazarev, Miller, and O'Bryant to correlated sumsets $A+B$ where B is correlated to A by the probabilities $\mathbb{P}(i \in B \mid i \in A) = p_1$ and $\mathbb{P}(i \in B \mid i \notin A) = p_2$. We provide some preliminary results towards finding $\mathbb{E}[|A+B|]$ and $\mathrm{Var}(|A+B|)$ using this framework.

Joint work with Hung Chu Viet, Noah Luntzlara, Thomas Martinez, Lily Shao, Chenyang Sun, Victor Xu, and Steven J. Miller.

(33) **Sandra Kingan**, Brooklyn College (CUNY)
Title: H-critical graphs
Abstract: We are interested in the class of 3-connected graphs with a minor isomorphic to a specific 3-connected graph H. A 3-connected graph is minimally 3-connected if deleting any edge destroys 3-connectivity. Suppose that G is a simple 3-connected graph with a simple 3-connected minor H. We say G is H-critical, if deleting any edge either destroys 3-connectivity or the H-minor. If H is minimally 3-connected, then G is also minimally 3-connected, and the class of H-critical graphs is the class of minimally

3-connected graphs with an H minor. In general, however, H is not minimally 3-connected, and in this case H-critical graphs are not minimally 3-connected graphs. Yet we have obtained splitter-type structural results for H-critical graphs that are very similar to Dawes' result on the structure of minimally 3-connected graphs. We also get a result that is very similar to Halin's bound on the size of minimally 3-connected graphs. I will present these results in this talk. The results are joint work with Joao Paulo Costalonga.

(34) **Sándor Kiss**, Hungary
Title: Sidon sets and bases
Abstract: Let $h \geq 2$ be an integer. We say a set A of nonnegative integers is an asymptotic basis of order h if every large enough positive integer can be written as a sum of h terms from A. The set of positive integers A is called an h-Sidon set if the number of representations of any positive integer as the sum of h terms from A is bounded by 1. In this talk I will speak about the existence of h-Sidon sets which are asymptotic bases of order $2h + 1$. This is a joint work with Csaba Sándor.

(35) **Jakub Konieczny**, Hebrew University of Jerusalem, Israel
Title: Automatic multiplicative sequences
Abstract: Automatic sequences – that is, sequences computable by finite automata – give rise to one of the most basic models of computation. As such, for any class of sequences it is natural to ask which sequences in it are automatic. In particular, the question of classifying automatic multiplicative sequences has attracted considerable attention in the recent years. In the completely multiplicative case, such classification was obtained independently by S. Li and O. Klurman and P. Kurlberg. The main topic of my talk will be the resolution of the general case, obtained in a recent preprint with M. Lemańczyk and C. Müllner. I will also discuss some early results on classification of automatic semigroups, which is the subject of ongoing work with O. Klurman.

(36) **Angel Kumchev**, Towson University
Title: Bounds for discrete maximal functions of codimension 3
Abstract: We study the bilinear discrete averaging operator $T_\lambda(f,g)(x) = \sum_{m,n \in V_\lambda} f(x-m)g(x-n)$, where f and g are functions in $\ell^p(\mathbb{Z}^d)$ and $\ell^q(\mathbb{Z}^d)$ and the summation is over the integer solutions $(m,n) \in \mathbb{Z}^{2d}$ of the equations

$$|m|^2 = |n|^2 = 2m \cdot n = \lambda,$$

where $|\cdot|$ is the standard Euclidean norm on $\mathbb{R}^d$. We prove an approximation formula for the Fourier multiplier of T_λ and establish the boundedness of the respective maximal operator from $\ell^p(\mathbb{Z}^d \times \ell^q(\mathbb{Z}^d)$ to $\ell^r(\mathbb{Z}^d)$ for a range of choices for p, q, r. Our work is related to classical work on simultaneous representations of integers by quadratic forms as well as to the study of point configurations in combinatorial geometry.

Joint work with T.C. Anderson and E.A. Palsson.

(37) **Jeffrey C. Lagarias**, University of Michigan
Title: Partial factorizations of products of binomial coefficients
Abstract: Let G_n denote the product of the binomial coefficients in the n-th row of Pascal's triangle. Then $\log G_n$ is asymptotic to $\frac{1}{2}n^2$ as $n \to \infty$. Let $G(n, x)$ denote the product of the maximal prime powers of all $p \leq x$ dividing G_n. We determine asymptotics of $\log G(n, \alpha n) \sim f(\alpha)n^2$ as $n \to \infty$, with error term. Here

$$f(\alpha) = \frac{1}{2} - \alpha \left\lfloor \frac{1}{\alpha} \right\rfloor + \frac{1}{2}\alpha^2 \left\lfloor \frac{1}{\alpha} \right\rfloor^2 + \frac{1}{2}\alpha^2 \left\lfloor \frac{1}{\alpha} \right\rfloor$$

for $0 < \alpha \leq 1$. The result is based on analysis of associated radix expansion statistics $A(n, x)$ and $B(n, x)$. The estimates relate to prime number theory, and vice versa.

Joint work with Lara Du.

(38) **Thai Hoang Lê**, University of Mississippi
Title: Additive bases in infinite abelian semigroups, II
This talk is a continuation of part I by Pierre-Yves Bienvenu, though it will be self-contained.

Let T be a semigroup and A be a basis T. If F is a finite subset of A and $A \setminus F$ is still a basis T (of a possibly different order), can we bound the order of $A \setminus F$ in terms of that of A and $|F|$? In the semigroup $\mathbf{N}$, this question was first studied by Erdős and Graham when F is a singleton, and by Nash and Nathanson for general F. We prove a general bound for all translatable semigroups. Besides studying the maximum order of $A \setminus F$, we also study its "typical" order.

Joint work with Pierre-Yves Bienvenu and Benjamin Girard.

(39) **Paolo Leonetti**, Università Bocconi, Italy
Title: On the density of sumsets
Abstract: We define a large family $\mathcal{D}$ of partial set functions $\mu : \mathrm{dom}(\mu) \subseteq \mathcal{P}(\mathbf{N}) \to \mathbf{R}$ satisfying certain axioms. Examples of "densities" $\mu \in \mathcal{D}$ include the asymptotic, Banach, logarithmic, analytic, Pólya, and Buck densities. We prove several results on sumsets which were previously obtained for the asymptotic density. For instance, we show that for each $n \in \mathbf{N}^+$ and $\alpha \in [0, 1]$, there exists $A \subseteq \mathbf{N}$ with $kA \in \mathrm{dom}(\mu)$ and $\mu(kA) = \alpha k/n$ for every $\mu \in \mathcal{D}$ and every $k = 1, \ldots, n$, where kA denotes the k-fold sumset $A + \cdots + A$.
Joint work with Salvatore Tringali.

(40) **Huixi Li**, University of Nevada, Reno
Title: On the connection between the Goldbach conjecture and the Elliott-Halberstam conjecture
Abstract: In this presentation we show that the binary Goldbach conjecture for sufficiently large even integers would follow under the assumptions of both the Elliott-Halberstam conjecture and a variant of the Elliott-Halberstam conjecture twisted by the Möbius function, provided that the sum of their level of distributions exceeds 1. This continues the work of Pan. An analogous result for the twin prime conjecture is obtained by Ram

Murty and Vatwani.
Joint work with Jing-Jing Huang.

(41) **Jared Duker Lichtman**, University of Oxford
Title: The Erdős primitive set conjecture
Abstract: A set of integers larger than 1 is called *primitive* if no member divides another. Erdős proved in 1935 that the sum of $1/(n \log n)$ over n in a primitive set A is universally bounded for any choice of A. In 1988, he famously asked if this universal bound is attained by the set of prime numbers. In this talk we shall discuss some recent progress towards this conjecture and related results, drawing on ideas from analysis, probability, and combinatorics.

(42) **Florian Luca**, University of the Witwatersrand, South Africa
Title: Prime factors of the Ramanujan τ-function
Abstract: Let $\tau(n)$ be the Ramanujan τ-function of n. In this talk, we prove some results about prime factors of $\tau(n)$ and its iterates. Assuming the Lehmer conjecture that $\tau(n) \neq 0$ for all n, we show that if n is even and $k \geq 1$, then $\tau^{(k)}(n)$ is divisible by a prime $p \geq 3^{k-1}+1$. Given a fixed finite set of odd primes $S = \{p_1, \ldots, p_\ell\}$, we give a bound on the number of solutions of n of the equation $\tau(n) = \pm p_1^{a_1} \cdots p_\ell^{a_\ell}$ for integers $a_1, \ldots, a_\ell$ and in case $S := \{3, 5, 7\}$, we show that there is no such $n > 1$.
Joint work with S. Mabaso and P. Stănică.

(43) **Noah Luntzlara**, University of Michigan
Title: Sets arising as minimal additive complements in the integers
Abstract: A subset C of a group G is a *minimal additive complement* to $W \subseteq G$
if $C + W = G$ and if $C' + W \neq G$ for any proper subset $C' \subsetneq C$. Work started by Nathanson has focused on which sets $W \subseteq \mathbb{Z}$ have minimal additive complements. We instead investigate which sets $C \subseteq \mathbb{Z}$ arise as minimal additive complements to some set $W \subseteq \mathbb{Z}$. We confirm a conjecture of Kwon in showing that bounded below sets containing arbitrarily large gaps arise as minimal additive complements. We provide partial results for determining which eventually periodic sets arise as minimal additive complements. We place bounds on the density of sets which arise as minimal additive complements to finite sets, including periodic sets which arise as minimal additive complements. We conclude with several conjectures and questions concerning the structure of minimal additive complements.
Joint work with Amanda Burcroff.

(44) **Ariane Masuda**, New York City College of Technology, CUNY
Title: Rédei permutations with cycles of length 1 and p
Abstract: Let $\mathbb{F}_q$ be the finite field of odd characteristic with q elements and $\mathbb{P}^1(\mathbb{F}_q) := \mathbb{F}_q \cup \{\infty\}$. Consider the binomial expansion $(x+\sqrt{y})^n = N(x,y) + D(x,y)\sqrt{y}$. For $n \in \mathbb{N}$ and $a \in \mathbb{F}_q$, the *Rédei function* $R_{n,a} \colon \mathbb{P}^1(\mathbb{F}_q) \to$

$\mathbb{P}^1(\mathbb{F}_q)$ is defined by

$$R_{n,a}(x) = \begin{cases} \dfrac{N(x,a)}{D(x,a)} & \text{if } D(x,a) \neq 0, x \neq \infty \\ \infty & \text{if } D(x,a) = 0, x \neq \infty, \text{ or if } x = \infty. \end{cases}$$

Rédei functions have been used in several applications such as cryptography and coding theory as well as in the generation of pseudorandom numbers and Pell equations. In this talk we will present results on Rédei permutations that decompose in cycles of length 1 and p, where p is prime. In particular, we will describe all Rédei functions that are involutions.

Joint work with Juliane Capaverde and Virgínia Rodrigues.

(45) **Nathan McNew**, Towson University
Title: Primitive sets in function fields
Abstract: A set of integers is *primitive* if no element divides another. Erdős showed that $f(A) = \sum_{a \in A} \frac{1}{a \log a}$ converges for any primitive set A of integers greater than one, and later conjectured this sum is maximized when A is the set P_1 of primes. Banks and Martin further conjectured that $f(\mathcal{P}_1) > \ldots > f(\mathcal{P}_k) > f(\mathcal{P}_{k+1}) > \ldots$, where $\mathcal{P}_j$ denotes the integers with exactly j prime factors. However, this was recently disproven by Lichtman. We consider the analogous questions for polynomials over a finite field $\mathbb{F}_q[x]$, obtaining bounds on the analogous sum, and find that while the analogue of the Banks and Martin conjecture similarly fails for small values of q, it seems likely to hold for larger values.

Joint work with Andrés Gómez-Colunga, Charlotte Kavaler and Mirilla Zhu.

(46) **Amanda Montejano**, Universidad Nacional Autónoma de México
Title: Zero-sum squares in bounded discrepancy $\{-1, 1\}$-matrices
Abstract: For $n \geq 5$, we prove that every $n \times n$ $\{-1, 1\}$-matrix $M = (a_{ij})$ with discrepancy $\text{disc}(M) = \sum a_{ij} \leq n$ contains a zero-sum square except for the diagonal matrix (up to symmetries). Here, a square is a 2×2 submatrix of M with entries $a_{i,j}, a_{i+s,s}, a_{i,j+s}, a_{i+s,j+s}$ for some $s \geq 1$, and the diagonal matrix is a matrix with all entries above the diagonal equal to -1 and all remaining entries equal to 1. In particular, we show that for $n \geq 5$ every zero-sum $n \times n$ $\{-1, 1\}$-matrix contains a zero-sum square.

Joint work with Edgardo Roldán-Pensado and Alma R. Arévalo.

(47) **Hamed Mousavi**, Georgia Tech
Title: A class of sums with unexpectedly high cancellation
Abstract: In this talk we report on the discovery of a general principle leading to an unexpected cancellation of oscillating sums, of which $\sum_{n^2 \leq x} (-1)^n e^{\sqrt{x-n^2}}$ is an example (to get the idea of the result). It turns out that sums in the class we consider are much smaller than would be predicted by certain probabilistic heuristics. After stating the motivation, we show a number of results in integer partitions. For instance we show a

"weak" version of pentagonal number theorem

$$\sum_{\ell^2<x} (-1)^\ell p(x-\ell^2) \ \sim \ 2^{-3/4}x^{-1/4}\sqrt{p(x)},$$

where $p(x)$ is the usual partition function.

Joint work with Ernie Croot.

(48) **Akshat Mudgal**, University of Bristol, UK
Title : Arithmetic combinatorics on Vinogradov systems
Abstract: In this talk, we consider the Vinogradov system of equations from an arithmetic combinatorial point of view. The number of solutions of this system, when the variables are restricted to a set of real numbers A, has been widely studied by researchers in both analytic number theory and harmonic analysis. In particular, there has been a significant amount of work regarding upper bounds for the number of solutions to the above system of equations. The objective of our talk will be of a different flavour, wherein we will try to address the following question: Given a set A with many solutions to the Vinogradov system, what other arithmetic properties can we infer about A?

(49) **Mel Nathanson**, Lehman College and CUNY Graduate Center
Title: Fundamental theorems in additive number theory
Abstract: Let A be a subset of the integers $\mathbf{Z}$, of the lattice $\mathbf{Z}^n$, or of any additive abelian semigroup X. The central problem in additive number theory is to understand the h-fold sumset

$$hA = \{a_1 + \cdots + a_h : a_i \in A \text{ for all } i = 1, \ldots, h\}.$$

If A is finite, what is the size of the sumset hA? If A is infinite, what is the density of hA? What is the structure of the sumset hA? Describe this for small h, and also asymptotically as $h \to \infty$. In how many ways can an element $x \in X$ be represented as the sum of h elements of A? For fixed r, what is the subset of hA consisting of elements that have at least r representations? Classical problems consider sums of squares, of kth powers, and of primes, but the general case is also important. This talk will discuss both old and very recent results about sumsets.

(50) **Kevin O'Bryant**, College of Staten Island and CUNY Graduate Center
Title: Rigorous proofs of stupid inequalities
Abstract: An inequality is *stupid* if it is true, but not for any particular reason. We will give a collection of techniques for proving stupid inequalities, each of which was useful in my recent work in explicit analytic number theory.

(51) **Péter Pál Pach**, MTA-BME Lendület (Momentum) Arithmetic Combinatorics Research Group, Budapest University of Technology and Economics, Hungary
Title: Counting subsets avoiding certain multiplicative configurations
Abstract: We will discuss results about enumerating subsets of $\{1, 2, \ldots, n\}$

avoiding certain multiplicative configurations. Namely, we will count primitive sets, h-primitive sets (where none of the elements divide the product of h other elements) and multiplicative Sidon sets. Most of these problems were raised by Cameron and Erdős.

Joint work with Hong Liu and Richárd Palincza.

(52) **Bhuwanesh Rao Patil**, PDF at IISER Berhampur, India
Title: Geometric progressions in syndetic sets
Abstract: In this talk, we will discuss the presence of arbitrarily long geometric progressions in syndetic sets, where a subset of $\mathbb{N}$ (the set of all natural numbers) is called *syndetic* if it intersects every set of l consecutive natural numbers for some natural number l. In order to understand it, we will explain the structure of the set $\{\frac{a}{b} \in \mathbb{N} : a, b \in A\}$ for a given syndetic set A.

(53) **Fei Peng**, Carnegie Mellon University
Title: Distribution of missing differences in diffsets
Abstract: Lazarev, Miller, and O'Bryant investigated the distribution of $|S+S|$ for S chosen uniformly at random from $\{0, 1, \ldots, n-1\}$, and proved the existence of a divot at missing 7 sums (the probability of missing exactly 7 sums is less than missing 6 or missing 8 sums). We study related questions for $|S-S|$, and show some divots from one end of the probability distribution, $P(|S-S| = k)$, as well as a peak at $k = 4$ from the other end, $P(2n-1-|S-S| = k)$. A corollary of our results is an asymptotic bound for the number of complete rulers of length n.

Joint with Scott Harvey-Arnold and Steven J. Miller.

(54) **Giorgis Petridis**, The University of Georgia
Title: A question of Bukh on sums of dilates
Abstract: There exists a $p < 3$ with the property that for all real numbers K and every finite subset A of a commutative group that satisfies $|A+A| \leq K|A|$, the dilate sum

$$A + 2 \cdot A = \{a + b + b : a, b \in A\}$$

has size at most $K^p|A|$. This answers a question of Bukh.

Joint work with Brandon Hanson.

(55) **Carl Pomerance**, Dartmouth College
Title: Symmetric primes
Abstract: Two odd primes p, q are said to form a symmetric pair if $|p-q| = \gcd(p-1, q-1)$, and we say a prime is symmetric if it belongs to some symmetric pair. The concept comes from a standard proof of quadratic reciprocity where one counts lattice points in the $p/2 \times q/2$ rectangle nestled in the first quadrant, both above and below the diagonal: p and q are a symmetric pair if and only if these counts agree. Over 20 years ago, Fletcher, Lindgren, and I showed that most primes are *not* symmetric, though the numerical evidence for this is very weak (only about 1/6 of the primes to 10^6 are asymmetric). In a new paper with Banks and Pollack we get a conjecturally tight upper bound for the distribution of symmetric primes

and we prove that there are infinitely many of them.

(56) **Wladimir Pribitkin**, College of Staten Island and CUNY Graduate Center
Title: Recounting partitions in memory of Freeman Dyson
Abstract: We shall present a short proof of Rademacher's famous formula for the partition function $p(n)$. Although the proof is old, its joint publication (with Brandon Williams) is not, and the communication that it engendered with Freeman Dyson is forever young. If time permits, we shall discuss a generalization to a broad class of functions.

(57) **Oliver Roche-Newton**, Johann Radon Institute for Computational and Applied Mathematics (RICAM) Linz, Austria
Title: Higher convexity and iterated sum sets
Abstract: An important generalisation of the sum-product phenomenon is the basic idea that convex functions destroy additive structure. This idea has perhaps been most notably quantified in the work of Elekes, Nathanson and Ruzsa, in which they used incidence geometry to prove that at least one of the sets $A + A$ or $f(A) + f(A)$ must be large.

I will discuss joint work with Hanson and Rudnev, in which we use a stronger notion of convexity to make further progress. In particular, we show that, if $A + A$ is sufficiently small and f satisfies this hyperconvexity condition, then we have unbounded growth for sums of $f(A)$. This in turn gives new results for iterated product sets of a set with small sum set.

(58) **Misha Rudnev**, University of Bristol, UK
Title: An update on the state-of-the-art sum-product inequality over the reals
Abstract: The aim of this somewhat technical talk is to clarify the underlying constructions and present a streamlined step-by-step self-contained proof of the sum-product inequality of Solymosi, Konyagin and Shkredov. The proof ends up with a slightly better exponent $4/3 + 2/1167$ than the previous world record.

Joint work with Sophie Stevens.

(59) **Javier Santiago**, University of Puerto Rico
Title: On permutation binomials of index $q^{e-1} + q^{e-2} + \cdots + 1$
Abstract: The permutation binomial $f(x) = x^r(x^{q-1} + A)$ was studied by K. Li, L. Qu, and X. Chen over $\mathbb{F}_{q^2}$. They found that for $1 \leq r \leq q + 1$, $f(x)$ is a permutation binomial if and only if $r = 1$. Over the finite field $\mathbb{F}_{q^3}$ of odd characteristic, X. Liu obtained an analogous result, in which for $1 \leq r \leq q^2 + q + 1$, $f(x)$ permutes $\mathbb{F}_{q^3}$ if and only if $r = 1$. In this investigation, we complete the characterization for $f(x)$ over both $\mathbb{F}_{q^2}$ and $\mathbb{F}_{q^3}$, as well as obtain a complete characterization over $\mathbb{F}_{q^4}$. Furthermore, for $e \geq 5$, we present some partial results which narrow down considerably the search for $r's$ that do indeed yield permutation binomials of the form $f(x) = x^r(x^{q-1} + A)$ over $\mathbb{F}_{q^e}$.

Joint work with Ariane Masuda and Ivelisse Rubio.

(60) **Wolfgang Schmid**, University of Paris 8, Saint-Denis, Paris
Title: Plus-minus weighted zero-sum sequences and applications to factorizations of norms of quadratic integers
Abstract: Let $(G,+)$ be a finite abelian group. A sequence $g_1,\dots,g_k$ over G is called a zero-sum sequence if $g_1+\cdots+g_k=0$ (we consider sequences that just differ by the ordering of the terms as equal). The concatenation of two zero-sum sequences is a zero-sum sequence and the set of all zero-sum sequences over G is thus a monoid. The arithmetic of these monoids has been the subject much investigation.

A sequence is called a *plus-minus weighted zero-sum sequence* if there is a choice of weights $w_i\in\{-1,+1\}$ such that $w_1g_1+\cdots+w_kg_k=0$. The set of all plus-minus weighted zero-sum sequences over G is a monoid as well. We present some results on the arithmetic of these monoids. Moreover, applications to factorizations of norms of quadratic integers are discussed.

Joint work with S. Boukheche, K. Merito and O. Ordaz.

(61) **James Sellers**, University of Minnesota, Duluth
Title: Garden of Eden partitions for Bulgarian and Austrian solitaire
Abstract: In the early 1980s, Martin Gardner popularized the game called Bulgarian Solitaire through his writings in Scientific American. After a brief introduction to the game, we will discuss a few results proven about Bulgarian Solitaire around the time of the appearance of Gardner's article and then quickly turn to the question of finding an exact formula for the number of Garden of Eden partitions that arise in this game. I will then introduce a related game known as Austrian Solitaire and consider a similar question about the Garden of Eden states that appear. The talk will be completely self-contained and should be accessible to a wide ranging audience. This is joint work with Brian Hopkins and Robson da Silva.

(62) **Steve Senger**, Missouri State University
Title: Point configurations determined by dot products
Abstract: Erdős' unit distance problem has perplexed mathematicians for decades. It asks for upper bounds on how often a fixed distance can occur in a large finite point set in the plane. We offer novel bounds on a family of variants of this problem involving multiple points, and relationships determined by dot products. Specifically, given a large finite set E of points in the plane, and a $(m\times m)$ matrix M of real numbers, we offer bounds on the number of m-tuples of points from E, $(x_1,x_2,\dots,x_m)$, satisfying $x_i\cdot x_j=m_{ij}$, the (i,j)th entry of M.

(63) **Oriol Serra**, Universitat Politècnica de Catalunya, Barcelona
Title: Extremal sets for Freiman theorem
Abstract: The well-known theorem of Freiman states that sets of integers with small doubling are dense subsets of d–dimensional arithmetic progressions. In connection with this theorem, Freiman conjectured a precise upper bound on the volume of a finite d–dimensional set A in terms of the

cardinality of A and of the sumset $A + A$. A set $A \subset \mathbb{Z}^d$ is d–dimensional if it is not contained in a hyperplane. Its volume is the smallest number of lattice points in the convex hull of a set B that is Freiman isomorphic to A. The conjecture is equivalent to saying that the extremal sets for this problem are long simplices, consisting of a d–dimensional simplex and an extremal 1–dimensional set in one of the dimensions. In this talk we will discuss a proof of the conjecture for a wide class of sets called chains. A finite set is a chain if there is an ordering of its elements such that initial segments in this ordering are extremal.

Joint work with G.A. Freiman.

(64) **George Shakan**, University of Oxford, UK
Title: An analytic approach to the cardinality of sumsets
Abstract: We describe some notions of additive structure that are useful for studying the Minkowski sum of discrete sets in large dimensions.

Joint work with Dávid Matolcsi, Imre Ruzsa, and Dmitrii Zhelezov.

(65) **Senia Sheydvasser**, CUNY Graduate Center
Title: A twisted Euclidean algorithm
Abstract: Considering that it is millennia old, it is surprising how useful the Euclidean algorithm still is and how often it yields new insights. In this talk, we will discuss an analog of the classical Euclidean algorithm which applies to rings equipped with an involution. We will show various applications of such an algorithm in number theory and geometry and potentially discuss some open problems.

(66) **I.D. Shkredov**, Steklov Mathematical Institute, Russia
Title: Growth in Chevalley groups and some applications
Abstract: Given a Chevalley group $\mathbf{G}(q)$ and a parabolic subgroup $P \subset \mathbf{G}(q)$, we prove that for any set A there is a certain growth of A relatively to P, namely, either AP or PA is much larger than A. Also, we study a question about intersection of A^n with parabolic subgroups P for large n. We apply our method to obtain some results on a modular form of Zaremba's conjecture from the theory of continued fractions and make the first step towards Hensley's conjecture about some Cantor sets with Hausdorff dimension greater than $1/2$.

(67) **Pablo Soberón**, Baruch College (CUNY)
Title: The topological Tverberg problem beyond prime powers
Abstract: Tverberg-type theory aims to establish sufficient conditions for a simplicial complex Σ such that every continuous map $f\colon \Sigma \to \mathbb{R}^d$ maps q points from pairwise disjoint faces to the same point in $\mathbb{R}^d$. Such results are plentiful for q a power of a prime. However, for q with at least two distinct prime divisors, results that guarantee the existence of q-fold points of coincidence are non-existent—aside from immediate corollaries of the prime power case. Here we present a general method that yields such results beyond the case of prime powers.

Joint work with Florian Frick.

(68) **Yonutz V. Stanchescu**, Afeka Academic College, Tel Aviv, Israel
Title: A proof of a structural result for small doubling sets in three dimensional Euclidean space
Abstract: We shall present the proofs of some best possible structural results for finite three-dimensional sets with a small doubling property.

(69) **Sophie Stevens**, Johann Radon Institute for Computational and Applied Mathematics (RICAM) Linz, Austria
Title: An update on the sum-product problem
Abstract: In new work with Misha Rudnev, we prove a stronger bound on the sum-product problem, showing that $\max(|A+A|, |AA|) \geq |A|^{\frac{4}{3}+\frac{2}{1167}-o(1)}$ for a finite set $A \subseteq \mathbb{R}$. This builds upon the work of Solymosi, Konyagin and Shkredov, although our paper is self-contained. I will give an overview of the arguments, both old and new, and describe some consequences of the new arguments.

(70) **Josiah Sugarman**, CUNY Graduate Center
Title: On the spectrum of the Conway-Radin Operator
Abstract: John Conway and Charles Radin introduced a hierarchical tiling of $\mathbf{R}^3$ they called a quaquaversal tiling. The orientations of these tiles exhibit rapid equidistribution not possible in two dimension. To study the distribution of these tiles Sadun and Draco analyzed the spectrum of the Hecke operator associated with this tiling. We shall discuss a few results and conjectures related to the spectrum of this operator.

(71) **Aled Walker**, Centre de Recherches Mathématiques, Montréal, , and Trinity College, Cambridge
Title: A tight structure theorem for sumsets
Abstract: In joint work with Andrew Granville and George Shakan, we show that for any finite set $A = \{0 = a_0 < a_1 < \cdots < a_{m+1} = b\}$ of integers, NA is as predicted whenever $N \geq b - m$, and that this bound is "best possible" in several families of cases.

(72) **Ethan White**, University of British Columbia
Title: Directions in $AG(2, p)$ and the clique number of Paley graphs
Abstract: The directions determined by a point set are the slopes of lines passing through at least two points of the set. A seminal result of Rédei tells us that at least $(p + 3)/2$ directions are determined by p points in $AG(2, p)$. We consider cartesian product point sets, i.e. a set of the form $A \times B \subset AG(2, p)$, where p is prime, A and B are subsets of $GF(p)$ each with at least two elements and $|A||B| < p$. In this case, we show that the number of directions determined is at least $|A||B| - \min\{|A|, |B|\} + 2$. This gives an upper bound of about $\sqrt{p/2}$ on the clique number of Paley graphs, matching a bound obtained by Hanson and Petridis last year. Our main tool is the use of the Rédei polynomial with Szőnyi's extension.

Joint work with József Solymosi and Daniel Di Benedetto.

(73) **Trevor Wooley**, Purdue University
Title: Condensation and densification for sets of large diameter
Abstract: Consider a set of integers A having finite diameter X, so that

$$\sup A - \inf A = X < \infty,$$

and a system of simultaneous polynomial equations $P_1(\mathbf{x}) = \ldots = P_r(\mathbf{x}) = 0$ to be solved with $\mathbf{x} \in A^s$. In many circumstances, one can show that the number $N(A; \mathbf{P})$ of solutions of this system satisfies $N(A; \mathbf{P}) \ll X^\epsilon |A|^\theta$ for a suitable $\theta < s$ and any $\epsilon > 0$. Such is the case with modern variants of Vinogradov's mean value theorem due to the author, and likewise Bourgain, Demeter and Guth. These estimates become worse than trivial when the diameter X is very large compared to $|A|$, or equivalently, when the set A is very sparse. This motivates the problem of seeking new sets of integers A' in a certain sense "isomorphic" to A having the property that (i) the diameter X' of A' is smaller than X, and (ii) the set A' preserves the salient features of the solution set of the system of equations $P_1(\mathbf{x}) = \ldots = P_r(\mathbf{x}) = 0$. We will report on our speculative meditations (both results and non-results) concerning this problem closely associated with the topic of Freiman homomorphisms.

(74) **Chi Hoi Yip**, University of British Columbia
Title: On the clique number of Paley graphs of prime power order
Abstract: Finding a reasonably good upper bound for the clique number of Paley graph is an old and open problem in additive combinatorics. A recent breakthrough by Hanson and Petridis using Stepanov's method gives an improved upper bound on $\mathbb{F}_p$, where $p \equiv 1 \pmod 4$. We extend their idea to the finite field $\mathbb{F}_q$, where $q = p^{2s+1}$ for a prime $p \equiv 1 \pmod 4$ and a non-negative integer s. We show the clique number of the Paley graph over $\mathbb{F}_{p^{2s+1}}$ is at most

$$\min\left(p^s\left\lceil\sqrt{\frac{p}{2}}\right\rceil, \sqrt{\frac{q}{2}} + \frac{p^s+1}{4} + \frac{\sqrt{2p}}{32}p^{s-1}\right).$$