CANT 2021: Zoom Conference

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Nineteenth Annual Workshop on
Combinatorial and Additive Number Theory
CUNY Graduate Center (via Zoom)
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Abstracts

(1) **George Andrews**, Pennsylvania State University
Email: gea1@psu.edu
Title: Schmidt Type partitions and modular forms
Abstract: In 1999, Frank Schmidt noted that the number of partitions of integers in which the first, third, fifth, etc. summands add to n is equal to p(n), the number of ordinary partitions of n. By invoking MacMahon's theory of Partition Analysis, we provide a context for this result which leads directly to many other theorems of this nature. Joint work with Peter Paule.

(2) **Gabriela Araujo-Pardo**, Universidad Nacional Autónoma de México, México
Email: garaujo@math.unam.mx
Title: Complete colorings on circulant graphs and digraphs
Abstract: A *complete* k-vertex-coloring of a graph G is a vertex-coloring of G using k colors such that for every pair of colors there is at least two incident vertices in G colored with this pair of colors. The *chromatic* $\chi(G)$ and *achromatic* $\alpha(G)$ numbers of G are the smallest and the largest number of colors in a complete proper k-vertex-coloring of G, therefore $\chi(G) \leq \alpha(G)$. The dichromatic number and the diachromatic number generalize the concepts of chromatic number and achromatic number. An *acyclic* k-vertex-coloring of a digraph D is vertex coloring using k colors such that D has no monochromatic cycles and a *complete* k-vertex-coloring of a digraph D is a vertex coloring using k colors such that for every ordered pair (i, j) of different colors, there is at least one arc (u, v) such that u has color i and v has color j. The dichromatic number $dc(D)$ and diachromatic number $dac(D)$ of D are the smallest and the largest number of colors in a complete proper k-vertex-coloring of D, and so $dc(D) \leq dac(D)$. We determine the achromatic and diachromatic numbers of some specific circulant graphs and digraphs and give general bounds for these two parameters on these graphs and digraphs. Also, we determine the achromatic index for circulant graphs of order $q^2 + q + 1$ using projective planes.
Joint work with Juan José Montellano-Ballesteros, Mika Olsen, and Christian Rubio-Montiel.

(3) **Louis-Pierre Arguin**, Baruch College (CUNY)
Email: Louis-Pierre.Arguin@baruch.cuny.edu
Title: The Fyodorov-Hiary-Keating conjecture
Abstract: In this talk, I will describe joint work with P. Bourgade and M. Radizwiłł to obtain precise estimates for the large values of the Riemann zeta function in a short interval of the critical line. Such results were first conjectured by Fyodorov, Hiary and Keating using random matrices and extreme values of stochastic processes. The proof of the conjecture is a mix of recent techniques in probability and of twisted moments in number theory, which I will explain.

(4) **Paul Baginski**, Fairfield University
Email: baginski@gmail.com
Title: Abundant numbers, semigroup ideals, and nonunique factorization
Abstract: A positive integer n is abundant if the sum of its divisors, $\sigma(n) = \sum_{d|n} d$, is greater than $2n$; n is perfect if $\sigma(n) = 2n$; and otherwise n is deficient. Both the set H of abundant numbers and the set H^* of non-deficient numbers are closed under multiplication, making them subsemigroups (in fact, semigroup ideals) of $(\mathbb{N}, \times)$. As a result, we can consider how elements of H^* (or H) factor into irreducible elements of H^* (resp. H), a concept related to Dickson's notion of primitive non-deficient number. As it turns out, non-deficient numbers (or abundant numbers) do not factor uniquely into products of irreducible non-deficient numbers (resp. irreducible abundant numbers). We describe the factorization theory of these two semigroups, showing that they possess rather extreme factorization behavior. These two semigroups are special cases of a more general phenomenon that occurs with semigroup ideals of factorial monoids. We will describe some of the algebraic and arithmetic theory of these semigroup ideals and mention some additional arithmetic functions that yield extreme factorization properties.

(5) **Emma Bailey**, University of Bristol, UK
Email: e.c.bailey@bristol.ac.uk
Title: Generalized moments and large deviations of random matrix polynomials and L-functions
Abstract: This talk will present results on generalised moments and large deviations for the random matrix groups associated with the symmetry classes for families of L-functions. Such a connection is motivated by the work of Keating and Snaith, Katz and Sarnak, etc. In particular, we formulate and prove results for a multiple contour integral representation (following Conrey et al.) of the moments of moments of $\zeta(1/2 + it)$. Additionally, motivated by a conjecture of Radziwiłł, we prove a large deviation result in the moderate regime for unitary characteristic polynomials. This talk includes work joint with Louis-Pierre Arguin, Theo Assiotis, and Jon Keating.

(6) **Jozsef Balogh**, University of Illinois at Urbana-Champaign
Email: jobal@illinois.edu
Title: On the lower bound on Folkman cube
Abstract: Folkman's Theorem asserts that for each k there exists a natural number $n = F(k)$ such that whenever the elements of $[n]$ are two-coloured, then there exists a set $A \subset [n]$ of size k with the property that all the sums of the form $\sum_{x \in B}$, where B is a nonempty subset of A, are contained in $[n]$ and have the same colour. In 1989, Erdős and Spencer showed that $F(k) \geq 2ck2/\log k$, where $c > 0$ is an absolute constant; here, we improve this bound significantly.
The Van der Waerden number $W(k, r)$ denotes the smallest n such that whenever $[n]$ is r–colored there exists a monochromatic arithmetic progression of length k. Similarly, the Hilbert cube number $h(k, r)$ denotes the smallest n such that whenever $[n]$ is r–colored there exists a monochromatic affine k–cube. We show a relation between the Hilbert cube number and the Van der Waerden number. The results are based on joint work with Sean Eberhard, Bhargav Narayanan, Mikhail Lavrov, George Shakan, Andrew Treglown,and Adam Zsolt Wagner.

(7) **Esther Banaian**, University of Minnesota
Email: banai003@umn.edu
Title: A generalization of Markov numbers
Abstract: One perspective of Markov numbers is that they are lengths of arcs in a triangulated, once-punctured torus where all arcs in the triangulation have length 1. We introduce a generalization of Markov numbers by instead considering the length of arcs in a triangulated sphere with one puncture and three orbifold points. This vantage point is inspired from the theory of cluster algebras from surfaces; however, no familiarity with cluster algebras will be assumed.

(8) **Valérie Berthé**, Université de Paris, CNRS, France
Email: berthe@irif.fr
Title: Dynamics of Ostrowski's numeration: Limit laws and Hausdorff dimensions
Abstract: Ostrowski's numeration allows the representation of integers and real numbers with respect to the convergents in the continued fraction expansion of a given real real number. This numeration has many applications from inhomogeneous Diophantine approximation to word combinatorics. We consider here this numeration according to a dynamical and probabilistic approach. We establish the existence of normal laws for the statistical properties of digits and we provide estimates on the Hausdorff dimension of sets of points whose Ostrowski expansions have restricted digits. These results are deduced from the spectral study of the associated transfer operators.
Joint work with Jungwon Lee.

(9) **Elżbieta Bołdyriew, John Haviland, Phúc Lâm, John Lentfer, Steven J. Miller, Fernando Trejos Suárez**,
Email: sjm1@williams.edu
Title: Completeness of generalized Fibonacci sequences
Abstract: A sequence of positive integers is complete if every positive integer can be expressed as a sum of its terms, using each term at most once. A positive linear recurrence sequence (PLRS) is a sequence defined by a homogeneous linear recurrence relation with nonnegative coefficients of the form $H_{n+1} = c_1 H_n + \cdots + c_L H_{n-L+1}$ and a particular set of initial conditions. We seek to determine when a PLRS is complete. Using combinatorial methods, we characterize completeness of several families based on their defining coefficients $c_1, \ldots, c_L$, and conjecture criteria for more general families. Our primary method is applying Brown's criterion, which says that an increasing sequence is complete if and only if the first term is 1 and each subsequent term is bounded above by the sum of all previous terms plus 1. We also find a more efficient way to check completeness. Specifically, the characteristic polynomial of any PLRS has exactly one positive root; by bounding the size of this root, the majority of sequences may be classified as complete or incomplete. Additionally, we show there exists an indeterminate region where the principal root does not reveal any information on completeness.

(10) **Peter Bradshaw**, University of Bristol
Email: peter.bradshaw@bristol.ac.uk
Title: Energy bounds for k-fold sums in very convex sets
Abstract: A set $A = \{a_1 < \cdots < a_N\}$ is considered convex (or 1-convex) if the adjacent differences $a_{i+1} - a_i$ form a monotone sequence. This property can be iterated to define s-convex sets for $s > 1$.

There is a maxim in additive combinatorics that convex sets are not additively structured, and the "more" convex a set, the less additive structure it exhibits. In this talk, we will establish a new energy bound which supports this intuition. Let $T_k(A)$ be the number of solutions to

$$a_1 + \cdots + a_k = a_{k+1} + \cdots + a_{2k},$$

where $a_i \in A$ for $1 \leq i \leq 2k$. We show, using purely elementary methods, that if A is an s-convex set and $k = 2^s$, then

$$T_k(A) \ll_k |A|^{2k-1-s+\alpha_s},$$

where $\alpha_s = \sum_{j=1}^{s} j 2^{-j}$. This essentially matches all known sumset bounds for s-convex sets.
Joint work with Brandon Hanson and Misha Rudnev.

(11) **Jörg Brüdern**, Universität Göttingen, Germany
Email: joerg.bruedern@mathematik.uni-goettingen.de
Title: Expander estimates for cubes
Abstract: Suppose that $\mathcal{A}$ is a subset of the natural numbers. The supremum α of all t with

$$\limsup N^{-t}\#\{a \in \mathcal{A} : a \leq N\} > 0$$

is the *exponential density* of $\mathcal{A}$.

We examine what happens if one adds a power to $\mathcal{A}$. Fix $k \geq 2$, and let β_k be the exponential density of

$$\{x^k + a : x \in \mathbb{N},\, a \in \mathcal{A}\}.$$

It is easy to see that $\beta_2 = \min(1, \frac{1}{2} + \alpha)$. One might guess that

$$\beta_k = \min(1, \frac{1}{k} + \alpha) \tag{$*$}$$

holds for all k, but we are far from a proof. All current world records for this problem are due to Davenport, and are 80 years old. In this interim report on ongoing work with Simon Myerson we describe a method for $k = 3$ that improves Davenport's results when $\alpha > 3/5$, and that confirms (*) in an interval $(\alpha_0, 1]$. A concrete value for α_0 will be released during the talk, and if time permits, we also discuss the perspectives to generalize the approach to larger values of k.

(12) **Scott Chapman**, Sam Houston State University
Email: STC008@SHSU.EDU
Title: When Is a Puiseux monoid atomic?
Abstract: A Puiseux monoid is an additive submonoid of the nonnegative rational numbers. If M is a Puiseux monoid, then the question of whether each nonunit element of M can be written as a sum of irreducible elements (that is, M is atomic) is surprisingly difficult. For instance, although various techniques have been developed over the past few years to identify subclasses of Puiseux monoids that are atomic, no general characterization of such monoids is known. Here we survey some of the most relevant aspects related to the atomicity of Puiseux monoids. We provide characterizations of when M is finitely generated, factorial, half-factorial, other-half-factorial, Prüfer, seminormal, root-closed, and completely integrally closed. In addition to the atomic property, precise characterizations are also not known for when M satisfies the ACCP, is a BF-monoid, or is an FF-monoid; in each of these cases, we construct classes of Puiseux monoids satisfying these properties.
Joint work with Felix Gotti and Marly Gotti.

(13) **Zachary Chase**, University of Oxford, UK
Email: zachman99323@gmail.com
Title: A random analogue of Gilbreath's conjecture
Abstract: An old conjecture of Gilbreath states that if you let $a_{0,n} = p_n$ be the n^{th} prime number and $a_{i,n} = |a_{i-1,n} - a_{i-1,n+1}|$ for $i, n \geq 1$, then $a_{i,1} = 1$ for all $i \geq 1$. We prove this conjecture when the primes are replaced by a suitable random sequence.

(14) **Shane Chern**, Pennsylvania State University
Email: shanechern@psu.edu
Title: Euclidean billiard partitions
Abstract: Euclidean billiard partitions are introduced by Andrews, Dragovic and Radnovic in their study of periodic trajectories of ellipsoidal billiards in the Euclidean space. They are integer partitions into distinct parts such that (E1) adjacent parts are never both odd; (E2) the smallest part is even. In this talk, I will discuss bivariate generating function identities that keep track of both the size and length not only for Euclidean billiard partitions but also for distinct partitions satisfying merely Condition (E1). In analogy, I also investigate distinct partitions such that adjacent parts are never both even.

(15) **Joshua Cooper**, University of South Carolina
Email: cooper@math.sc.edu
Title: Recurrence ranks and moment sequences
Abstract: We introduce the "moment rank" and "unitary rank" of numerical sequences, close relatives of linear-recursive order. We show that both parameters can be characterized by a broad set of criteria involving moments of measures, types of recurrence relations, Hankel matrix factorizations, Waring rank, analytic properties of generating functions, and algebraic properties of polynomial ideals. In the process, we solve the "complex finite-atomic" and "integral finite-atomic" moment problems: Which sequences arise as the moments of a finite-atomic complex/integer-valued measures on $\mathbb{C}$?
Joint work with Grant Fickes.

(16) **Danielle Cox**, Mount Saint Vincent University, Canada
Email: danielle.cox@msvu.ca
Title: A sequence arising from diffusion in graphs
Abstract: In the chip-firing variant, diffusion chips placed on the vertices of a finite, simple graph are redistributed during this dynamic process by flowing from areas of high concentration to low concentration. In this talk we will study a sequence arising from the diffusion process on complete graphs and explore a connection to polyominoes.
Joint work with Todd Mullen (University of Saskatchewan) and Richard Nowakowski (Dalhousie University).

(17) **Michael Curran**, University of Oxford, UK
Email: Michael.Curran@maths.oxford.ac.uk
Title: Sumset structure, size, and Ehrhart theory
Abstract: Given a finite subset A of $\mathbb{Z}^d$, Khovanskii proved the remarkable result that for sufficiently large h the size of the h-fold sumset hA is a polynomial in h. Recently, Granville and Shakan showed that there is also a simple description of the structure of hA for large h. I will discuss connections with the h-fold sumset and Ehrhart's theory of counting lattice points in polytopes. In joint work with Leo Goldmakher, we use these connections to give the first effective bounds on "sufficiently large" in the theorems of Khovanskii and Granville-Shakan for $d \geq 2$, assuming that the convex hull of A is a simplex.

(18) **Anne de Roton**, Université de Lorraine, France
Email: anne.de-roton@univ-lorraine.fr
Title: Critical sets with small sumset in $\mathbb{R}$
Abstract: For A and B subsets of real numbers, the sum of A and B is defined as the set $A + B = \{a + b, a \in A, b \in B\}$. If A and B are measurable bounded sets in $\mathbb{R}$ and if λ is the inner Lebesgue measure, the classical lower bound $\lambda(A + B) \geq \lambda(A) + \lambda(B)$ was improved by I. Ruzsa in 1991. He gave a lower bound for $\lambda(A+B)$ in terms of $\lambda(A)$, $\lambda(B)$, the diameter of A and the ratio $\lambda(A)/\lambda(B)$. In this talk we shall give some consequences of Ruzsa's inequality and describe the critical sets for which the lower bound is attained or almost attained.

(19) **Robert Donley**, Queensborough Community College (CUNY)
Email: rdonley@qcc.cuny.edu
Title: Vandermonde convolution for ranked posets
Abstract: Pascal's triangle counts the number of paths between opposing corners in a rectangular grid. The classical Chu-Vandermonde identity follows by factoring paths through a certain fixed set of points in the grid. The collection of these identities may then be interpreted as a model in terms of the representation theory of the Lie algebra $sl(2)$. Examples of ranked posets with similar behavior are considered.

(20) **Robert Dougherty-Bliss**, Rutgers University - New Brunswick
Email: robert.w.bliss @gmail.com
Title: More irrationally good approximations from Beukers integrals
Abstract: Given that almost all reals are irrational, it is a cruel irony that establishing the irrationality of any *naturally occurring* constant is notoriously difficult. While we *know*, morally, that $\zeta(5)$, Catalan's constant G, and the Euler–Mascheroni constant γ are irrational, we are mostly unable to *prove* such things. To make some headway we systematically tweaked the integrals used by Frits Beukers in his famous rendition of Apéry's theorem that $\zeta(3)$ is irrational. This search produced many promising *candidates* for irrationality proof, such as $\sqrt{\pi}\Gamma(7/3)/\Gamma(-1/6)$ and $\Gamma(19/6)/(\sqrt{\pi}\Gamma(8/3))$, and we hope to entice our audience into exploring these and others.

(21) **Artūras Dubickas**, Vilnius University, Lithuania
Email: arturas.dubickas@mif.vu.lt
Title: On polynomial Sidon sequences
Abstract: It seems likely that there exists a polynomial with integer coefficients $f(x)$ for which $f(1), f(2), f(3), \ldots$ is a Sidon sequence, but no such polynomial is known. (One of the candidates is $f(x) = x^5$.) In the opposite direction it is known that no linear or a quadratic integer polynomial at consecutive integer values forms a Sidon sequence. The linear case is trivial, while the quadratic case, as observed by Ruzsa, follows by a density result of Erdős. The same is true for the polynomial $f(x) = x^4$ by a result of Swinnerton-Dyer (1968). In 2001 Ruzsa constructed an "almost" polynomial Sidon sequence. He also conjectured that no *cubic* polynomial with integer coefficients at consecutive integer points generates a Sidon sequence. In joint work with Aivaras Novikas (Vilnius), we proved this conjecture (to appear in *Mathematische Nachrichten*).

(22) **Sean Eberhard**, University of Cambridge, UK
Email: eberhard@maths.com.ac.uk
Title: The apparent structure of dense Sidon sets
Abstract: Mostly I will survey dense Sidon sets. I will explain a joint observation (it cannot be called joint work), with Freddie Manners, that all known dense (i.e., $1 - o(1)$ times square-root size) Sidon subsets of abelian groups arise from projective planes through an explicit construction generalizing Singer's. It can be shown that no further examples arise from Desarguesian planes in this way, but there are some strange examples arising from non-Desarguesian planes, well-known to finite geometers for decades but apparently little known among additive combinatorialists. In any case we conjecture that every dense Sidon set is related to some projective plane in this manner. I will also survey some smaller examples. As the density is decreased, there are a great variety of interesting examples, but all of some algebraic nature.

(23) **Shalom Eliahou**, Université du Littoral Côte d'Opale, France
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Title: Optimal bounds on the growth of iterated sumsets in abelian semigroups
Abstract: A classical theorem by Macaulay in 1927 gives necessary and sufficient conditions for a sequence $(d_i)_{i\geq 0}$ of positive integers to be the Hilbert function of a standard graded algebra. In a recent work, we used the *necessity* of Macaulay's conditions to provide good upper bounds on the growth of iterated sumsets in abelian semigroups, better than those given by Plünnecke's inequality. In this talk, we show that these upper bounds are in fact *sharp*, in the sense that they are actually attained by suitable subsets of suitable abelian semigroups. The construction of these optimizing subsets is based on the *sufficiency* of Macaulay's conditions and on Gröbner bases techniques.
Joint work with Eshita Mazumdar.

(24) **Christian Elsholtz**, Graz University of Technology, Austria
Email: elsholtz@math.tugraz.at
Title: Fermat's Last Theorem Implies Euclid's infinitude of primes
Abstract: We show that Fermats last theorem (for any fixed exponent $n \geq 3$) and a combinatorial theorem of Schur on monochromatic solutions of $a + b = c$ implies that there exist infinitely many primes. Similarly, we discuss implications of Roth's theorem on arithmetic progressions, Hindman's theorem, and infinite Ramsey theory toward Euclid's theorem.

As a consequence we see that Euclid's theorem is a necessary condition for many interesting (seemingly unrelated) results in number theory and combinatorics.

(25) **Jinhui Fang**, Nanjing University of Information Science and Technology, China
Email: fangjinhui1114@163.com
Title: On generalized perfect difference sumset
Abstract: Let $\mathbb{Z}$ be the set of integers and $\mathbb{N}$ be the set of positive integers. For a nonempty set A of integers and any integers n, $h \geq 2$, denote $r_{A,h}(n)$ by the number of representations of n of the form $n = a_1 + a_2 + \cdots + a_h$, where $a_1 \leq \cdots \leq a_h$ and $a_i \in A$ for $i = 1, 2, \cdots, h$ and $d_A(n)$ by the number of (a, a') with $a, a' \in A$ such that $n = a - a'$. The set A of integers is called *a perfect difference sumset* if $r_{A,2}(n) = 1$ for all integers n and $d_A(n) = 1$ for all positive integers n. Recently, we considered generalized perfect difference sumsets and proved that, if two functions $f_1 : \mathbb{N} \to \mathbb{N}$ and $f_2 : \mathbb{Z} \to \mathbb{N}$ satisfy that $\liminf_{u\to\infty} f_1(u) \geq 2$ and $\liminf_{|u|\to\infty} f_2(u) \geq 2$, then there exists a set A of integers such that: (i) $d_A(n) = f_1(n)$ for all $n \in \mathbb{N}$ and $r_{A,2}(n) = f_2(n)$ for all $n \in \mathbb{Z}$; (ii) $\limsup_{x\to\infty} A(-x, x)/\sqrt{x} \geq 1/\sqrt{2}$. Furthermore, following Cilleruelo and Nathanson's work in [European J. Combin. 34 (2013), 1297-1306], we proved that there exists a set A of integers such that: (i) $r_{A,3}(n) = 2$ for all $n \in \mathbb{Z}$ and $d_A(n) = 1$ for all $n \in \mathbb{N}$; (ii) $A(x) \gg x^{\sqrt{5}-2+o(1)}$.

(26) **Leonid Fel**, Technion - Israel Institute of Technology, Israel
Email: lfel@cv.technion.ac.il
Title: Genera of numerical semigroups and polynomial identities for degrees of syzygies
Abstract: We derive a polynomial identities of arbitrary degree n for syzygies degrees of numerical semigroups $S_m = \langle d_1, \ldots, d_m \rangle$ and show that for $n \geq m$ they contain higher genera $G_r = \sum_{s\in\mathbb{Z}_>\setminus S_m} s^r$ of semigroup. We find a minimal number $g_m = B_m - m + 1$ of algebraically independent genera G_r and equations, related any of $g_m + 1$ genera, where $B_m = \sum_{k=1}^{m-1} \beta_k$ and β_k denote the total and partial Betti numbers of S_m. The number g_m decreases with a growth of semigroup's symmetry and reaches a value $m - 2$ when S_m becomes a complete intersection.

(27) **Amanda Francis**, Mathematical Reviews, AMS
Email: amandafrancis@gmail.com
Title: Sequences of integers related to resistance distance in structured graphs
Abstract: In this talk I will consider the numbers $T(G)$ of spanning trees and $F_{u,v}(G)$ of spanning 2-forests that separate vertices u and v in a graph G; these numbers are related to the electrical resistances between nodes of a network. I will discuss the integer sequences $\{T(G_n)\}$ and $\{F_{u,v}(G_n)\}$ for certain families $\{G_n\}$ of highly structured simple graphs. These sequences exhibit interesting properties, including some related to monotonicity and growth rate.

(28) **Zoltan Furedi**, University of Illinois at Urbana-Champaign
Email: z-furedi@illinois.edu
Title: An upper bound on the size of Sidon sets
Abstract: Combining two elementary proofs, we decrease the gap between the upper and lower bounds by 0.2% in a classical combinatorial number theory problem. We show that the maximum size of a Sidon subset of $\{1, 2, , n\}$ is at most $n^{1/2} + 0.998n^{1/4}$ for sufficiently large n.
Joint work with József Balogh and Souktik Roy (UIUC).

(29) **Mikhail Gabdullin**, Steklov Mathematical Institute, Russia
Email: gabdullin.mikhail@yandex.ru
Title: Sets whose differences avoid squares modulo m
Abstract: Let $A \subset \mathbb{Z}_m$ be such that $A - A$ does not contain nonzero quadratic residues modulo m. It is highly believed that for square-free m the bound $|A| \ll_\varepsilon m^\varepsilon$ holds for any $\varepsilon > 0$, but this hypothesis seems to be far beyond the reach of current methods. M. Matolcsi and I. Ruzsa proved that $|A| \leq m^{1/2}$ if m is square-free and has prime divisors of the form $4k + 1$ only, and I showed that $|A| \ll m^{1/2+o(1)}$ for almost all positive integers m. In our joint work with Kevin Ford we overcome this square-root barrier and prove that if $\varepsilon(m) \to 0$ arbitrarily slowly, then for almost all m we have $|A| \leq m^{1/2-\varepsilon(m)}$.

(30) **Krystian Gajdzica**, Jagiellonian University, Krakow, Poland
Email: krystian.gajdzica@im.uj.edu.pl
Title: Arithmetic properties of the restricted partition function $p_{\mathcal{A}}(n, k)$
Abstract: Let $\mathcal{A} = (a_n)_{n\in\mathbb{N}_+}$ be a sequence of positive integers. The function $p_{\mathcal{A}}(n, k)$ counts the number of multi-color partitions of n into parts in $\{a_1, \ldots, a_k\}$. We examine several arithmetic properties of the sequence $(p_{\mathcal{A}}(n, k) \pmod{m})_{n\in\mathbb{N}}$ for an arbitrary fixed integer $m \geqslant 2$, and apply them to the special cases of $\mathcal{A}$. In particular, for a fixed parameter k, we investigate both the upper bound for the odd density of $p_{\mathcal{A}}(n, k)$ and the lower bound for the density of $\{n \in \mathbb{N} : p_{\mathcal{A}}(n, k) \not\equiv 0 \pmod{m}\}$. Furthermore, we present some new results related to restricted m-ary partitions and state a few open questions at the end of the talk.

(31) **Kåre Schou Gjaldbæk**, CUNY
Email: hardkxre@hotmail.com
Title: Classification of quadratic packing polynomials on sectors of $\mathbb{R}^2$
Abstract: Define the sector $S(\alpha) := \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq \alpha x\}$. A function which bijectively maps $S(\alpha)$ onto $\mathbb{N}_0$ is called a *packing function* or, in case of a polynomial, a *packing polynomial*. We classify all quadratic packing polynomials in case of rational α. This generalizes results of Stanton, Nathanson, and Fueter and Pólya. Given the non-existence of quadratic packing polynomials on sectors with irrational α, it provides a complete classification of quadratic packing polynomials for sectors $S(\alpha)$ of $\mathbb{R}^2$. The existence of higher degree packing polynomials is an open problem.
Joint work with Madeline Brandt.

(32) **Daniel G. Glasscock**, University of Massachusetts, Lowell
Email: daniel_glasscock@uml.edu
Title: Sums and intersections of multiplicatively invariant sets in the integers
Abstract: In the late 60s, Harry Furstenberg made a number of conjectures concerning the relationship between restricted digit Cantor sets (and their dynamical generalizations) with respect to different bases on the real line. He showed that one of his conjectures implies a positive answer to a question in the integers frequently attributed to Erdős: is it true that for finitely many n, the digit 7 is required to write the number 2^n in decimal? That conjecture of Furstenberg remains open, but two of his conjectures were recently resolved in works of Hochman, Shmerkin, and Wu. In this talk, Ill explain what we can deduce from these recent revelations about sums and intersections of restricted digit sets (and their dynamical generalizations) in the integers. A first example: in the interval $\{1, ..., N\}$, there are $o(N^{\epsilon})$ many integers that avoid the digit 1 in base 3 and avoid the digits 1, 2, 3, 4, and 5 in base 7.
Joint work with Joel Moreira (U. of Warwick) and Florian K. Richter (Northwestern U.).

(33) **David Grynkiewicz**, University of Memphis
Email: diambri@hotmail.com
Title: Characterizing infinite subsets of lattice points having finite-like behavior.
Abstract: A lattice Λ is a discrete subgroup of $\mathbb{R}^d$, meaning every bounded subset of $\mathbb{R}^d$ contains only finitely many lattice points. The canonical example is the subgroup $\mathbb{Z}^d \subseteq \mathbb{R}^n$. This talk regards the general question of which infinite subsets G_0 of a lattice $\Lambda \subseteq \mathbb{R}^d$ nonetheless still possess finite-like behavior. We will briefly overview some of the main ideas used to make this precise, including the use of zero-sum sequences, primitive partition identities, and the well-studied notion of elasticity. A finite sequence S of terms from G_0 is zero-sum if the sum of its terms is zero, and it is called a minimal zero-sum sequence if it cannot be partitioned into two proper, nontrivial zero-sum subsequences. A partition of the zero-sum sequence S

into nontrivial zero-sum subsequences $T_1, T_2, \dots, T_k$ is viewed as a factorization of S, and the elasticity of S is then the ratio between the maximal number k of factors that can occur in a factorization of S and the minimum number k of potential factors. The elasticity of G_0 is the supremum over the elasticities of all possible zero-sum sequences S with terms from G_0. This talk will briefly go over portions of the recent characterization of when G_0 has finite elasticity in terms of the combinatorial geometry of convex cones generated by elements from G_0, focusing on several combinatorial and geometric aspects in broad terms.

(34) **Lajos Hajdu**, University of Debrecen, Hungary
Email: hajdul@science.unideb.hu
Title: Multiplicative (in)decomposability of polynomial sequences
Abstract: In the talk we present various results concerning the multiplicative (in)decomposability of value sets of integer polynomials. We give a complete description of polynomials whose value set in $\mathbb{N}$ is totally multiplicatively primitive, and we provide results for the case where some elements of the value sets can be omitted. In the quadratic case our results into this direction are sharp. Further, we give a multiplicative analogue of a result of Sárközy and Szemerédi concerning *changing* elements of the set of shifted k-th powers $\{1+1, 2^k+1, \dots, x^k+1, \dots\}$ (related to a conjecture of Erdős), which is nearly sharp. In our proofs we combine several tools from Diophantine number theory, a classical theorem of Wiegert on the number of divisors of positive integers and a theorem of Bollobás on the Zarankiewicz function, related to extremal bipartite graphs.
Joint work with A. Sárközy.

(35) **Norbert Hegyvári**, Eötvös University and Rényi Institute, Hungary
Email: hegyvari@renyi.hu
Title: Communication complexity, coding, and combinatorial number theory
Abstract: In this talk we collect some problems from combinatorial number theory which are related to coding theory and communication complexity. In the first part, a special pseudo-recursive sequence $A_\alpha := \{a_n = \lfloor 2^{n-1}\alpha \rfloor : n = 1, 2, \dots\}$ (which was proposed by Rényi, Erdős, and Graham) will be used for an encryption algorithm. We will have a discussion on the completeness of sets in $\mathbb{N}^k$ higher dimension, and the connection with communication complexity. Finally we will describe a test, also related to communication complexity, which decides that $f(A)g(B) = h(C)$ for thin sets A, B, C in $\mathbb{F}_p$.

(36) **Harald Andres Helfgott**, Universität Göttingen, Germany
Email: harald.helfgott@gmail.com
Title: Expansion, divisibility and parity
Abstract: We will discuss a graph that encodes the divisibility properties of integers by primes. We show that this graph is shown to have a strong local expander property almost everywhere. We then obtain several consequences in number theory, beyond the traditional parity barrier. For example, for the Liouville function λ (that is, the completely multiplicative function with $\lambda(p) = -1$ for every prime), we have $(1/\log x) \sum_{n \leq x} \lambda(n)\lambda(n+1)/n = O(1/\sqrt{\log\log x})$, which is stronger than a well-known result by Tao. We also manage to prove, for example, that $\lambda(n+1)$ averages to 0 at almost all scales when n restricted to have a specific number of prime divisors $\Omega(n) = k$, for any "popular" value of k (that is, $k = \log\log N + O(\sqrt{\log\log N})$ for $n \leq N$). We will give a quick overview of the combinatorial ideas behind the proof. Joint work with M. Radziwill.

(37) **Russell Jay Hendel**, Towson University
Email: RHendel@towson.edu
Title: Sums of squares: Methods for proving identity families
Abstract: This paper presents both a method and a result. The result presents a closed formula for the sum of the first $m+1, m \geq 0$, squares of the sequence $F^{(k)}$ where each member is the sum of the previous k members and with initial conditions of $k-1$ zeroes followed by a 1. The generalized result includes the known result of sums of squares of the Fibonacci numbers and a recent result of Schumaker on sums of squares of Tribonacci numbers. To prove the identities uniformly for all k, the Algebraic Verification method is presented which reduces proof of an identity to verification of the equality of finitely many pairs of finite-degree polynomials, possibly in several variables. Several other papers proving families of identities are examined, and it is suggested that the collection of the uniform proof methods used in these papers could produce a new trend in stating and proving identities.

(38) **Neil Hindman**, Howard University
Email: nhindman@aol.com
Title: Strongly image partition regular matrices
Abstract: A $u \times v$ matrix A with rational entries is *image partition regular over* $\mathbb{N}$ provided that whenever $\mathbb{N}$ is finitely colored, there exists $\vec{x} \in \mathbb{N}^v$ such that the entries of $A\vec{x}$ are monochromatic. We say that A is *strongly image partition regular over* $\mathbb{N}$ provided that for every IP-set C in $\mathbb{N}$ there exists $\vec{x} \in \mathbb{N}^v$ such that the entries of $A\vec{x}$ are in C. (An IP set is a set containing all finite distinct sums from an infinite sequence.). Many characterizations of image partition regular matrices are known. We provide here two sufficient conditions and one necessary condition for a matrix with rank u to be strongly image partition regular and show that. such matrices can be expanded horizontally at will. We provide several examples showing that our results are sharp. Joint work with Dona Strauss.

(39) **Robert Hough**, SUNY at Stony Brook
Email: robert.hough@stonybrook.edu
Title: Subconvexity of the Shintani zeta functions
Abstract: Shintani introduced zeta functions enumerating class numbers of binary cubic forms, which I recently generalized to include an automorphic form evaluating the "shape" of the form. I will discuss recent joint work with Eun Hye Lee which proves a subconvex estimate for these zeta functions in the critical strip. The proofs combine Bhargava's averaging method of enumerating forms with traditional approaches to subconvexity including van der Corput's inequality.

(40) **Ayesha Hussain**, University of Bristol, UK
Email: ayesha.hussain@bristol.ac.uk
Title: Distributions of Dirichlet character sums
Abstract: Over the past few decades, there has been a lot of interest in partial sums of Dirichlet characters. Montgomery and Vaughan showed that these character sums remain a constant size on average and, as a result, a lot of work has been done on the distribution of the maximum. In this talk, we will investigate the distribution of these character sums themselves, with the main goal being to describe the limiting distribution as the prime modulus approaches infinity. This is motivated by the work of Kowalski and Sawin on Kloosterman paths.

(41) **Alex Iosevich**, University of Rochester
Email: iosevich@gmail.com
Title: Uniform distribution and incidence theorems
Abstract: We are going to discuss some connections between uniform distribution of sequences, Erdős-Tuŕan theorems, and Szemerédi-Trotter type incidence theory.

(42) **Brad Isaacson**, New York City College of Technology (CUNY)
Email: bissacson@citytech.cuny.edu
Title: Three imprimitive character sums
Abstract: We express three imprimitive character sums in terms of generalized Bernoulli numbers. These sums are generalizations of sums introduced and studied by Arakawa, Berndt, Ibukiyama, Kaneko and Ramanujan in the context of modular forms and theta function identities. As a corollary, we obtain a formula for cotangent power sums considered by Apostol.

(43) **Yifan Jing**, University of Illinois at Urbana-Champaign
Email: yifanjing17@gmail.com
Title: Minimal and nearly minimal measure expansions in connected locally compact groups
Abstract: Let G be a connected unimodular group equipped with a Haar measure μ, and suppose $A, B \subseteq G$ are nonempty and compact. An inequality by Kemperman gives us $\mu(AB) \geq \min\{\mu(A) + \mu(B), \mu(G)\}$. We obtain characterizations of G, A, and B such that the equality holds, answering a question asked by Kemperman in 1964. We also get near equality versions

of the above results with a sharp exponent bound for connected compact groups. This confirms conjectures made by Griesmer and by Tao and can be seen as a Freiman $3k-4$-theorem up to a constant factor for this setting. Joint work with Chieu-Minh Tran.

(44) **Fatma Karaoglu**, Tekirdag Namik Kemal University, Turkey
Email: fkaraoglu@nku.edu.tr
Title: On the number of lines of a smooth cubic surface
Abstract: A cubic surface is an algebraic variety of degree three in projective three space. The number of lines on a cubic surface is an important geometric invariant. This number depends on the field. If the field is algebraically closed, a smooth cubic surface has 27 lines. Otherwise, several possibilities arise. By revisiting the Cayley-Salmon proof on 27 lines, we realised that there is a certain polynomial whose number of roots give this counting. In this talk, we are interested in the smooth cubic surfaces with 27, 15, 9 and 3 lines over a non-algebraically closed field. These are the interesting cases since the condition on the number of lines depends on the number of roots of a certain polynomial over the field.

We will describe two new normal forms for smooth cubic surfaces with at least 9 lines, each of which involves either 4 or 6 parameters over the given field. Up to isomorphism, any smooth cubic surface with at least 9 lines arises in this way. Using birational maps, the rational points and lines on these normal forms will be described explicitly.

The starting point of this work was experimental. The experiments lead us to make conjectures and give idea to prove them. The proofs are computer-free.

(45) **Mizan R. Khan**, Eastern Connecticut State University
Email: KHANM@easternct.edu
Title: To count clean triangles we count on *imph*
Abstract: A clean lattice triangle in $\mathbb{R}^2$ is a triangle that does not contain any lattice points on its sides other than its vertices. The central goal of this paper is to count the number of clean triangles of a given area up to unimodular equivalence. In doing so we use a variant of the Euler phi function which we call $imph(n)$ (imitation phi). Nothing in this article is original. In addition to Paul Scott's work, almost certainly all of this material is squirrelled away in some of Bruce Reznick's papers. However, our fervent hope is that after listening to this talk members of the audience will try to discover a suitable arithmetic function called *oomph* and a suitable class of arithmetic functions called *oops*.
Joint work by Mizan R. Khan and Riaz R. Khan

(46) **Sándor Kiss**, Budapest University of Technology and Economics, Hungary
Email: ksandor@math.bme.hu
Title: Generalized Sidon sets of perfect powers
Abstract: For $k \geq 2$ and an infinite set of positive integers A, let $R_{A,k}(n)$ denote the number of representations of the positive integer n as the sum of k distinct terms from A. Given positive integers $g \geq 1$, $h \geq 2$, we say a set of positive integers A is a $B_h[g]$ set if every positive integer can be written as the sum of h not necessarily distinct terms from A at most g different ways. A set A of positive integers is called a basis of order k if every positive integer can be written as the sum of k terms from A. A few years ago, V. H. Vu proved the existence of a thin basis of order k formed perfect powers. In my talk I would like to speak about $B_h[g]$ sets formed by perfect powers. I prove the existence of a set A formed by perfect powers with almost possible maximal density such that $R_{A,h}(n)$ is bounded. The proof is based on the probabilistic method.
Joint work with Csaba Sándor.

(47) **Oleksiy Klurman**, University of Bristol, UK
Emali: lklurman@gmail.com
Title: On the "variants" of the Erdős discrepancy problem
Abstract: The famous Erdős discrepancy problem (now theorem of Tao) asserts that for any sequence $\{a_n\}_{n\geq 1} = \{\pm 1\}^{\mathbb{N}}$

$$\sup_{n,d} \left| \sum_{k=1}^{n} a_{kd} \right| = +\infty.$$

It was observed during the Polymath 5 project that the direct analog of this statement over the polynomial ring $\mathbb{F}_q[x]$ is false. In this talk, I plan to discuss several different forms of the EDP over $\mathbb{F}_q[x]$ and explain some surprising features that are not present in the number field setting.
Joint work A. Mangerel (CRM) and J. Teravainen (Oxford).

(48) **Sergei Konyagin**, Steklov Mathematical Institute, Moscow, Russia
Email: konyagin@mi-ras.ru
Title: Gaps between totients
Abstract: We study the set $\mathcal{D}$ of positive integers d for which the equation $\phi(a) - \phi(b) = d$ has infinitely many solution pairs (a, b). We show that $\min \mathcal{D} \leq 154$, exhibit a specific A so that every multiple of A is in $\mathcal{D}$, and show that any progression $a \mod d$ with $4|a$ and $4|d$, contains infinitely many elements of $\mathcal{D}$. We also show that the Generalized Elliott-Halberstam Conjecture implies that $\mathcal{D}$ contains all positive, even integers.
Joint work with Kevin Ford.

(49) **Emmanuel Kowalski**, Eidgenössische Technische Hochschule Zürich, Switzerland
Email: kowalski@math.ethz.ch
Title: Some families of Sidon sets arising in algebraic geometry
Abstract: We will report on a a simple construction of apparently new Sidon sets in finite abelian groups, arising from the embedding of an algebraic curve of genus at least 2 in its jacobian. The talk will explain all the necessary background in concrete terms. We will also discuss some connections with previously known constructions, as well as applications to equidistribution of certain exponential sums, and raise some questions about these examples. Joint work with A. Forey and J. Fresán.

(50) **Noah Kravitz**, Princeton University
Email: nkravitz@princeton.edu
Title: Inverse problems for minimal complements
Abstract: Given a subset W of an abelian group G, a subset C is called an additive complement for W if $W + C = G$; if, moreover, no proper subset of C has this property, then we say that C is a minimal complement for W. Early work in this area focused on determining which sets W have minimal complements, particularly in the setting $G = \mathbb{Z}$. Kwon later initiated the study of the "inverse problem", namely, determining which subsets C can arise as minimal complements for some W. In this direction, our main result is that in a finite abelian group G, all "small" subsets arise as minimal complements: more precisely, every non-empty subset C of size $|C| \leq |G|^{1/3}/(5\log|G|)^{2/3}$ is a minimal complement for some W.
Joint work with Noga Alon and Matt Larson.

(51) **Jeffrey Lagarias**, University of Michigan
Email: lagarias@umich.edu
Title: Partial factorizations of a generalized product of binomial coefficients
Abstract: Let G_n denote the product of the binomial coefficients in the n-th row of Pascal's triangle. It is easy to show that $\log G_n$ is asymptotic to $\frac{1}{2}n^2$ as $n \to \infty$. Let $G(n,x)$ denote the product of the maximal prime powers of all $p \leq x$ dividing G_n. Previous work (with Lara Du) determined asymptotics of $\log G(n, \alpha n) \sim f(\alpha)n^2$ as $n \to \infty$, with error term. Here $f(\alpha)$ is an interesting function, given by $f(\alpha) = \frac{1}{2} + \frac{1}{2}\alpha^2\lfloor\frac{1}{\alpha}\rfloor^2 + \frac{1}{2}\alpha^2\lfloor\frac{1}{\alpha}\rfloor - \alpha\lfloor\frac{1}{\alpha}\rfloor$ for $0 < \alpha \leq 1$. This result was obtained by analysis of associated radix expansion statistics $A(n,x)$ and $B(n,x)$ associated to the radix expansions of integers up to n to prime radix bases p over $p \leq n$. The size of the remainder term of the estimates relates to distribution of primes, and conversely. The present work studies a generalization, called $\overline{\overline{G}}_n$, of these products in which the radix expansion statistics are taken over all radix bases $b \leq n$. In this case $\log \overline{\overline{G}}_n$ is asymptotic to $\frac{1}{2}n^2 \log n$. Defining $\overline{\overline{G}}(n,x)$ as above, the asymptotics of $\log \overline{\overline{G}}(n, \alpha n)$ are determined to be of the form $f(\alpha)n \log n + g(\alpha)n$ with an (unconditional) power-savings remainder term in n. A generalized binomial coefficient interpretation of this integer sequence is described. Joint work with Lara Du and Wijit Yangit (U. Mich).

(52) **Thái Hoàng Lê**, University of Mississippi
Email: leth@olemiss.edu
Title: Bohr sets in sumsets
Abstract: A Bohr set in an abelian group G is a subset of the form

$$B(K,\epsilon) = \{g \in G : |\chi(g) - 1| < \epsilon \, \forall \chi \in K\}$$

where K is a finite subset of the dual group $\widehat{G}$. A classical theorem of Bogolyubov says that if $A \subset \mathbf{Z}$ has positive upper density δ, then $A+A-A-A$ contains a Bohr set $B(K,\epsilon)$ where $|K|$ and ϵ depend only on δ. While the same statement for $A-A$ is not true (a result of Kříž), Bergelson and Ruzsa proved that if $r+s+t=0$, then $rA+sA+tA$ contains a Bohr set (here $rA = \{ra : a \in A\}$). I will discuss this phenomenon in compact abelian groups (which include all finite abelian groups, and in particular $\mathbf{Z}_N$ and $\mathbf{F}_p^n$), and an analogue of Bergelson-Ruzsa's result where rA, sA, tA are replaced by images of A under certain continuous homomorphisms of G. I will also discuss an analogue result for partitions. This talk is based on ongoing work with Anh Lê.

(53) **Noah Lebowitz-Lockard**,
Email: nlebowi@gmail.com
Title: On factorizations into distinct parts
Abstract: Let $f(n)$ be the number of representations of n as an unordered product of integers greater than 1 and $F(n)$ the number of representations in which the parts are distinct. (These products are also called "factorizations" or "multiplicative partitions".) Starting with Oppenheim's asymptotic formula for the sum of $f(n)$ over all $n \leq x$ in 1927, we discuss various theorems related to factorizations from the past century. We also discuss recent bounds for $F(n)$. Finally, we show that $f(n)/F(n)$ is typically very small with respect to n, in the process resolving a 40 year old problem originally set forth by Canfield, Erdős, and Pomerance.

(54) **Paolo Leonetti**, Università Bocconi, Milano, Italy
Email: leonettipaolo@gmail.com
Title: On Poissonian pair correlation sequences with few gaps
Abstract: A sequence (x_n) on $[0,1)$ is said to have Poissonian pair correlation if

$$\forall s > 0, \quad \lim_{N\to\infty} \frac{1}{N} \# \left\{ 1 \leq i \neq j \leq N : \|x_i - x_j\| \leq \frac{s}{N} \right\} = 2s.$$

It is known that, if (x_n) has Poissonian pair correlations, then the number $g(n)$ of different neighboring gap lengths of $\{x_1, \ldots, x_n\}$ satisfies $\lim_n g(n) = \infty$. First, we show that, if (x_n) has Poissonian pair correlations, then the maximum among the multiplicities of the neighboring gap lengths of $\{x_1, \ldots, x_n\}$ is $o(n)$ as $n \to \infty$, improving the previous result. Then, we prove that for every function $f : \mathbf{N}^+ \to \mathbf{N}^+$ with $\lim_n f(n) = \infty$ there exists a sequence (x_n) with Poissonian pair correlations such that $g(n) \leq f(n)$ for all sufficiently large n, answering negatively a question posed by G. Larcher.

(55) **Anqi Li**, MIT
Email: anqili@mit.edu
Title: Local properties of difference sets
Abstract: Erdős and Shelah asked what we can learn about a large and complicated object X from properties that are satisfied by each small piece of X. We study the following variant of this problem, first studied by Erdős and Sós. Given a set of real numbers A, we consider the *difference set* $A - A = \{|a-b| : a, b \in A\}$. While a random set A is expected to have $|A - A| = \Theta(|A|^2)$, arithmetic progressions satisfy $|A - A| = \Theta(|A|)$. Let $g(n, k, \ell)$ denote the minimum size of $|A - A|$, taken over all sets A of n numbers that satisfy the following local property: every subset $A' \subset A$ of k numbers satisfies $|A' - A'| \geq \ell$. Intuitively, every k numbers from A span many differences. We derive several new bounds for $g(n, k, \ell)$. Erdős and others were interested in *linear thresholds* of local properties problems: the smallest ℓ for which the size of the global property is superlinear. We establish the linear threshold of the differences problem.

Theorem 1. For every k, we have $g(n, k, k-1) = n-1$ and $g(n, k, k) \gg n$.

Theorem 2. When k is a power of two, we have

$$g\left(n, k, \frac{k^{\log_2(3)}+1}{2}\right) = \Omega\left(n^{1+\frac{1}{k-1}}\right).$$

This is the simplest of a family of bounds that we derive.

(56) **Richard Magner**, Boston University
Email: rmagner@bu.edu
Title: Classifying partition regular polynomials in a nonlinear family
Abstract: A polynomial equation over $\mathbb{Z}$ is said to be partition regular if for every finite partition of $\mathbb{Z}$ there exists a cell containing a (nonzero) solution. A classic theorem of Schur states that $x+y=z$ is partition regular. Rado's Theorem then gives a full classification for when linear polynomials are partition regular. Later, Csikvári, Gyarmati, and Sárközy have showed that the nonlinear equation $x + y = z^2$ is not partition regular and asked if the equation $x + y = wz$ is partition regular. V. Bergelson and N. Hindman independently answered this question in the positive. We generalize this result by investigating for which constants (a, b, c, n, m) the equation $ax + by = cw^m z^n$ is partition regular, with a near complete classification on the coefficients for when this happens. Of particular interest in this talk will be how some algebraic number theory gets used to establish lack of partition regularity in some cases.
Joint work with Sohail Farhangi.

(57) **Mehdi Makhul**, Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austria
Email: mehdi.makhul@oeaw.ac.at
The Elekes-Szabó problem and the uniformity conjecture
Abstract: Finding upper bounds for the number of intersections of the zero set of a polynomial $F \in \mathbb{F}[x, y, z]$ and a Cartesian product $A \times B \times C$, under certain non-degeneracy conditions, is called the Elekes-Szabó problem. In this paper we give a conditional improvement to the Elekes-Szabó problem

over the rationals, assuming the Uniformity Conjecture. Our main result states that for $F \in \mathbb{Q}[x,y,z]$ belonging to a particular family of polynomials, and any finite sets $A, B, C \subset \mathbb{Q}$ with $|A| = |B| = |C| = n$, we have

$$|Z(F) \cap (A \times B \times C)| \ll n^{2-\frac{1}{s}}.$$

The value of the integer s is dependent on the polynomial F, but is always bounded by $s \leq 5$, and so even in the worst applicable case this gives a quantitative improvement on a bound of Raz, Sharir and de Zeeuw.

We give several applications to problems in discrete geometry and arithmetic combinatorics. For instance, for any set $P \subset \mathbb{Q}^2$ and any two points $p_1, p_2 \in \mathbb{Q}^2$, we prove that at least one of the p_i satisfies the bound

$$|\{\|p_i - p\| : p \in P\}| \gg |P|^{3/5},$$

where $\| \cdot \|$ denotes Euclidean distance. This gives a conditional improvement to a result of Sharir and Solymosi.
Joint work with Oliver Roche-Newton, Sophie Stevens, and Audie Warren.

(58) **Brian McDonald**, University of Rochester
Email: bmcdonald3879@gmail.com
Title: Cycles of arbitrary length in distance graphs on $\mathbb{F}_q^d$
Abstract: For $E \subseteq \mathbb{F}_q^d$, $d \geq 2$, where $\mathbb{F}_q$ is the finite field with q elements, we consider the distance graph $\mathcal{G}_t^{dist}(E)$, $t \neq 0$, where the vertices are the elements of E, and two vertices x, y are connected by an edge if $||x-y|| \equiv (x_1-y_1)^2 + \cdots + (x_d-y_d)^2 = t$. We prove that if $|E| \geq C_k q^{\frac{d+2}{2}}$, then $\mathcal{G}_t^{dist}(E)$ contains a statistically correct number of cycles of length k. We are also going to consider the dot-product graph $\mathcal{G}_t^{prod}(E)$, $t \neq 0$, where the vertices are the elements of E, and two vertices x, y are connected by an edge if $x \cdot y \equiv x_1y_1 + \cdots + x_dy_d = t$. We obtain similar results in this case using more sophisticated methods necessitated by the fact that the function $x \cdot y$ is not translation invariant. The exponent $\frac{d+2}{2}$ is improved for sufficiently long cycles.
Joint work with Alex Iosevich and Gail Jardine.

(59) **Karyn McLellan** , Mount Saint Vincent University, Canada
Email: KarynMcLellan@msvu.ca
Title: A problem on generating sets containing Fibonacci numbers
Abstract: At the Sixteenth International Conference on Fibonacci Numbers and Their Applications the following problem was posed: Let S be the set generated by these rules: Let $1 \in S$ and if $x \in S$, then $2x \in S$ and $1 - x \in S$, so that S grows in generations: $G_1 = \{1\}$, $G_2 = \{0, 2\}$, $G_3 = \{-1, 4\}$, ... Prove or disprove that each generation contains at least one Fibonacci number or its negative. We will show that every integer k can be found in some G_i and will disprove the above statement by finding an expression for the generation index i for any given k. We will use a variety of recurrence sequences including the dragon curve sequence, properties of binary numbers, and a computer calculation to find numerous counterexamples. Joint work with Danielle Cox.

(60) **Akshat Mudgal**, University of Bristol, UK
Email: am6393@bristol.ac.uk
Title : Additive energies on spheres
Abstract: In this talk, we will study additive properties of lattice points on spheres in four dimensions. Thus, letting A be a set of lattice points on a sphere of radius N in $\mathbb{R}^4$, we use $E_s(A)$ to denote the number of solutions to the equation

$$x_1 + \cdots + x_s = x_{s+1} + \cdots + x_{2s},$$

where $x_1, \ldots, x_{2s} \in A$. These additive energies were studied by Bourgain and Demeter in relation to the discrete restriction conjecture on spheres, who proved that $E_2(A) \ll_\epsilon N^\epsilon |A|^{2+1/3}$ using purely incidence geometric methods. In our talk, we present a threshold breaking result in this direction, thus showing that

$$E_2(A) \ll_\epsilon N^\epsilon |A|^{2+1/3-c},$$

for some small $c > 0$. This involves utilising multiple ideas from combinatorial geometry as well as arithmetic combinatorics.

(61) **Karameh Muneer**, Palestine Polytechnic University, Palestine
Email: muneerk@ppp.edu
Title: Generalizations of B.Berggren and Price matrices
Abstract: The aim of this paper is to generalize B.Berggren and Price matrices, in finding Pythagorean primitive triples by using the matrices method. They used the matrices of power one to do so. I generalized the method by using any power of matrices to find Pythagorean primitive triples.

(62) **Mel Nathanson**, Lehman College (CUNY)
Email: melvyn.nathanson@lehman.cuny.edu
Title: Sidon systems for linear forms and the Bose-Chowla argument
Abstract: A classical Sidon set of order h is a set A of integers for which there is a bound on the number of representations of any integer as the sum of h elements of A. Equivalently, the number of representations is bounded with respect to the linear form $x_1 + x_2 + \cdots + x_h$. There is an analogous definition of a Sidon set with respect to an arbitrary linear form $c_1x_1 + c_2x_2 + \cdots + c_hx_h$. This talk will describe recent results about Sidon sets for linear forms.

(63) **Lan Nguyen**, University of Wisconsin - Parkside
Email: nguyenl@uwp.edu
Title: On the existence of bi-Lipschitz equivalences and quasi-isometries between arithmetic metric spaces with word metrics and the local-global principle
Abstract: Large scale geometric structures of spaces and the algebraic structures of groups influence each other and understanding such connections has important consequences and is a part of Gromov's program. In this presentation, I will discuss some problems which are at the interface of number theory, metric geometry and geometric group theory and recent progress in solving them. The central topics of these problems are the existence of

quasi-isometries and bi-Lipschitz equivalences between the arithmetic metric spaces arising from the additive group of integers with respect to different infinite integer generating bases which were first raised by Richard E. Schwartz and then generalized by Melvyn B. Nathanson and the local-global principle for these maps.

(64) **Konstantin Olmezov**, Moscow Institute of Physics and Technology, Russia
Email: fractalonkolm@gmail.com
Title: On additive energy of convex sets with higher concavity
Abstract: A set (or a strictly increasing sequence) $A = \{a_1 < \cdots < a_n\} \subset \mathbb{R}$ is *convex* if the sequence of differences $\Delta A := (a_{i+1} - a_i)_{i=1}^{n-1}$ is strictly increasing, or, equivalently, $\Delta^2 A := \Delta(\Delta A) > 0$. In 2000 Konyagin proved that $E(A) \leq |A|^{5/2}$ for any convex A using incidence geometry. Garaev obtained the same result by a pure combinatorial method. Later many new results about convexity were obtained using the incidence approach. In particular, Shkredov in 2013 proved that $E(A) \leq |A|^{32/13}$.

We analyze Garaev's proof and improve the main lemma to obtain two strong partial bounds

- if $\Delta^2 A > 0$, $\Delta^3 A \leq 0$ then $E(A) \ll |A|^{12/5}$
- if $\Delta^2 A > 0$, $\Delta^3 A < 0$, $\Delta^4 A \leq 0$ then $E(A) \ll |A|^{7/3}$

where $\Delta^k A := \Delta(\Delta^{k-1} A),\ k \geq 2$. More precisely, we obtain corresponding bounds for the number $J_l(A)$ of solutions of the equation

$$a_{i+l} - a_i = a_{j+k} - a_j,\ i < j,\ k > 0.$$

(65) **Péter Pál Pach**, TU Budapest, Hungary
Email: p.p.pach@gmail.com
Title: Sum-full sets are not zero-sum-free
Abstract: Let A be a finite, nonempty subset of an abelian group. We show that if every element of A is a sum of two other elements, then A has a nonempty zero-sum subset. That is, a (finite, nonempty) sum-full subset of an abelian group is not zero-sum-free.
Joint work with Seva Lev and János Nagy.

(66) **Jianping Pan**, University of California, Davis
Email: jpgpan@ucdavis.edu
Title: Tableaux and polynomial expansions
Abstract: I will talk about several polynomials arising from Schubert calculus, that are generating functions of various kinds of tableaux. I will introduce the basic concepts with examples, and how polynomials expansions are manifested via uncrowding algorithms on the tableaux.

(67) **Bhuwanesh Rao Patil**, IIT Roorkee, India
Email: bhuwanesh1989@gmail.com
Title: Multiplicative patterns in syndetic sets
Abstract: A subset A of the natural numbers $\mathbb{N}$ is called syndetic if there exists $l \in \mathbb{N}$ such that A intersects every set of l consecutive natural numbers. In 2006, Beiglböck et al. raised the question of whether or not a syndetic set contains arbitrarily long geometric progressions. In 2019, Glasscock et al. investigated an infinite family of syndetic sets containing arbitrarily long geometric progressions.

In this talk, we describe some new infinite families of syndetic sets containing long geometric progressions. Moreover, we deduce some combinatorial properties of a syndetic set A from the set R(A) of integral ratios of elements in A.

(68) **Arthur Paul Pedersen**, City College (CUNY)
Email: apedersen@cs.ccny.cuny.edu
Title: The Hahn-Hölder
Abstract: Established is an extension of Hans Hahn's classic embedding theorem for ordered Abelian groups to embeddings into ordered formal power series fields, thereby completing the analogy to Otto Hölder's Theorem for Archimedean ordered groups. A basic economic application: Any coherent, weakly ordered system of preferences admits representation by a linear utility form whose codomain is a totally ordered field extension of the real numbers wrought of the system's relational signature. Likewise, the technical developments delineate key ingredients of an unconditionally full account of subjective expected utility in the style of Savage and Anscombe and Aumann. Features enjoyed by these fields will thereupon be examined in connection with the theory of real-closed fields. Time permitting, an impossibility theorem is shown to curb the scope of these embeddings vis--vis Archimedeanness.

(69) **Paul Pollack**, University of Georgia
Email: pollack@math.uga.edu
Title: Multiplicative orders mod p
Abstract: I will survey what is known about the distribution of the orders of integers mod p, as p varies. Particular attention will be paid to problems of the following sort: For fixed a and b, how do the order of a mod p and the order of b mod p compare, as p varies? The proofs will draw from the elementary, algebraic, and analytic strands of number theory. (So hopefully something for everyone!)

(70) **Sean Prendiville**, University of Lancaster, UK
Email: s.prendiville@lancaster.ac.uk
Title: Extremal Sidon sets are Fourier uniform, with arithmetic applications
Abstract: Erdős and Freud proved that the largest Sidon subset of an interval of integers is equidistributed in sub-intervals. Analogously, Lindström proved equidistribution in sub-progressions. Generalising both results, we prove equidistribution in Bohr sets. We establish this by showing that extremal Sidon sets are pseudorandom, as measured by their Fourier transform. As an application, we show that in any finite colouring of an extremal Sidon set there is a monochromatic solution to the equation $x+y+z+w = v$.
Joint work with Miquel Ortega (Universitat Politcnica de Catalunya).

(71) **Pooja Punyani**, Indian Institute of Technology, New Delhi, India.
Email: poojapunyani.pp@gmail.com
Title: On characterizing small changes in the Frobenius number
Abstract: For any set of positive integers A with $\gcd(A) = 1$, let $\Gamma(A)$ denote the set of integers that are expressible as a linear combination of elements of A with non-negative integer coefficients. Then $\mathsf{g}(A)$ denote the *largest* positive integer not in $\Gamma(A)$. Formula for $\mathsf{g}(A)$ is well known for 2-set $\{a, b\}$. We determine small values in the set $\{\mathsf{g}(a, b) - \mathsf{g}(a, b, c) : c \in \mathbb{N}\}$ and characterize c for which small changes are attained.
Joint work with Amitabha Tripathi.

(72) **Yaghoub Rahimi**, Georgia Institute of Technology
Email: yaghoub.rahimi@gatech.edu
Title: Endpoint ℓ^p improving estimates for prime averages
Abstract: In this talk our focus is on the averages along the prime numbers and the ℓ^p improving type inequalites. Consider the average sum

$$A_N f(x) = \frac{1}{N} \sum_{n=1}^{N} f(x-n)\Lambda(n).$$

We prove sharp ℓ^p improving for these averages, and sparse bounds for the maximal function $\sup_N |A_N f|$. The simplest inequality is that for $f = \mathbf{1}_F$ and $g = \mathbf{1}_G$ with $F, G \subset I$ there holds

$$\langle A_N f, g \rangle \leq C \langle f \rangle_I \langle g \rangle_I \times \psi(\langle f \rangle_I \langle g \rangle_I),$$

where we have

$$\psi(t) = \begin{cases} 1 + |\log(t)| & \text{assuming GRH} \\ (1 + |\log(t)|)^2 & \text{otherwise.} \end{cases}$$

Assuming the Generalized Riemann Hypothesis (GRH), these inequalities are sharp. The proof depends upon the circle method, and an interpolation argument of Bourgain. Use of smooth numbers in the proof enabled us to get these bounds.
Joint work with Michael T. Lacey and Hamed Mousavi.

(73) **Alex Rice**, Millsaps College
Email: arice2386@gmail.com
Title: Two constructions related to well-known distance problems
Abstract: Erdős famously asked: Given $n \in \mathbf{N}$ what is the minimum number of distinct distances determined by n points in a plane? In a continuous setting, Bourgain (among others, independently) showed that if a subset of the plane has positive upper density, then it determines all sufficiently large distances. Here, we discuss two constructions: one, a precise (as opposed to asymptotic) potential resolution of the Erdős question in all dimensions for the ℓ^1 or *taxicab* metric, and the other, a general construction exhibiting that Bourgain's result cannot be quantitatively improved. The former includes joint work with six (at the time) Millsaps College undergraduate students: Vajresh Balaji, Olivia Edwards, Anne Marie Loftin, Solomon Mcharo, Lo Phillips, and Bineyam Tsegaye. The latter answers a question posed by Alex Iosevich at CANT 2019, held in honor of Jean Bourgain.

(74) **Sinai Robins**, University of Sao Paolo, Brazil
Email: sinai.robins@gmail.com
Title: The null set of a of a polytope, and the Pompeiu property for polytopes
Abstract: Proving various facts that may at first appear to get further and further away from traditional number theory, we study the null set $N(P)$ of the Fourier transform of a polytope P in $\mathbf{R}^d$. We find that this null set does not contain (almost all) circles in $\mathbf{R}^d$. As a consequence, the null set does not contain the algebraic varieties $\{z \in \mathbf{C}^d \mid z_1^2 + \cdots + z_d^2 = \alpha\}$, for each fixed $\alpha \in \mathbf{C}$. Hence we get an explicit proof that the Pompeiu property is true for all polytopes. Our proof uses the Brion-Barvinok theorem in combinatorial geometry, together with some properties of the Bessel functions. The original proof that polytopes (as well as other bodies) possess the Pompeiu property was given by Brown, Schreiber, and Taylor (1973) for dimension 2. In 1976, Williams observed that the same proof also works for $d > 2$ and, using eigenvalues of the Laplacian, gave another proof valid for $d \geq 2$ that polytopes indeed have the Pompeiu property.
Joint work with Fabrcio Caluza Machado.

(75) **Oliver Roche-Newton**, Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austria
Email: o.rochenewton@gmail.com
Title: The Elekes-Szabo Theorem and sum-product estimates for sparse graphs
Abstract: The Elekes-Szabo Theorem gives an upper bound for the size of the intersection of a Cartesian product and a non-degenerate polynomial surface. This theorem has had many beautiful applications in discrete geometry. In this talk, I will discuss a recent application giving new information in sum-product theory. In particular, I will discuss new results for sum-product type estimates restricted to sparse graphs.

(76) **Ryan Ronan**, Baruch College (CUNY)
Email: ryan.p.ronan@gmail.com
Title: An asymptotic for the growth of Markoff-Hurwitz tuples
Abstract: For integer parameters $n \geq 3$, $a \geq 1$, and $k \geq 0$ the Markoff-Hurwitz equation is the diophantine equation

$$x_1^2 + x_2^2 + \cdots + x_n^2 = ax_1x_2 \cdots x_n + k.$$

In this talk, we establish an asymptotic count for the number of integral solutions with $\max\{x_1, x_2, \ldots, x_n\} \leq R$. When $n = a = 3$ and $k = 0$ this equation is known simply as the Markoff equation, for which the asymptotic count was studied in detail by Zagier in 1982. The previous best result for $n \geq 4$ is due to Baragar in 1998 who established an exponential rate of growth with exponent $\beta(n) > 0$ when $k = 0$, and which is not, in general, an integer. We use methods from symbolic dynamics to improve this asymptotic count, and which yield a new interpretation of this exponent β as the unique parameter for which there exists a certain conformal measure on projective space. Joint work with Alex Gamburd and Michael Magee.

(77) **Souktik Roy**, University of Illinois at Urbana-Champaign
Email: souktik2@illinois.edu
Title: Generalized sums and products
Abstract: Using tools from semialgebraic/o-minimal geometry, we prove that for two bivariate polynomials $P(x, y)$ and $Q(x, y)$ with coefficients in $\mathbb{R}$ or $\mathbb{C}$ to simultaneously exhibit small expansion, they must exploit the underlying additive or multiplicative structure of the field in nearly identical fashion. This in particular generalizes a result of Shen and yields an Elekes–Rónyai type structural result for symmetric nonexpanders. Our result aims to place sum-product phenomena into a more general picture of model-theoretic interest.
Joint work with Yifan Jing and Chieu-Minh Tran.

(78) **Misha Rudnev**, University of Bristol, UK
Email: misharudnev@gmail.com
Title: On distinct values of bilinear forms, cross-ratios, etc.
Abstract: My favourite open problem in discrete geometry asks, over sets Q of N points in the plane and a non-degenerate bilinear form ω, for the minimum cardinality of the set

$$\omega(Q) = \{\omega(q, q') : \, q, q' \in Q\}.$$

If ω is symmetric, these are pairwise dot products, if skew-symmetric – areas of triangles, rooted at the origin (assuming that all of Q does not lie on a single line through the origin).

The best known bounds are just slightly better than $N^{2/3}$ if ω is skew-symmetric: roughly $\omega(Q) \geq N^{.7}$ over the reals and $\omega(Q) \geq N^{.67}$ in positive characteristic p, sufficiently large, relative to N. Otherwise, only the easy Szemerédi-Trotter type bound $\omega(Q) \gg N^{2/3}$ has been proven. In fact, a weaker point-plane incidence theorem suffices, and the bound $\omega(Q) \gg N^{2/3}$ also holds in positive characteristic, for $N < p^{3/2}$.

In the skew-symmetric case, the small quantitative improvement over this comes from relating the putative adverse scenario to another wide open question of the minimum number of distinct cross-ratios, defined by a N-element subset of the projective line.

(79) **Imre Z. Ruzsa**, Alfréd Rényi Institute of Mathematics, Hungary
Email: ruzsa@renyi.hu
Title: Additive decomposition of square-free numbers
Abstract: Ostmann's "inverse Goldbach" problem is whether the set of primes is asymptotically a sumset. We consider the analogous question for the set S of square-free numbers. We cannot decide if there are sets A and B, each with more than one element, such that $S\Delta(A+B)$ is finite (Δ = symmetric difference). However, a weaker approximation by sumsets is possible.

Theorem 1. *For every k there are infinite sets $A_1, \dots, A_k \subseteq \mathbf{N} \cup \{0\}$ such that $A_1 + \dots + A_k \supseteq S$ and*

$$d\big((A_1 + .. + A_k) \setminus S\big) = 0.$$

For an inner approximation we have the following weaker result:

Theorem 2. *For every $\varepsilon > 0$ there are infinite sets $A, B \subseteq \mathbf{N}$ such that $A + B \subset S$ and $d_L(A) > 6/\pi^2 - \varepsilon$.*

The situation is simpler if we allow negative numbers. Put

$$S' = S \cup -S = \{\text{signed squarefree numbers}\}.$$

Theorem 3. *For every k there are infinite sets $A_1, \dots, A_k \subset \mathbf{Z}$ such that $A_1 + \dots + A_k = S'$.*

(80) **Carlo Sanna**, Politecnico di Torino, Italy
Email: carlo.sanna.dev@gmail.com
Title: Additive bases and Niven numbers
Abstract: Let $g \geq 2$ be an integer. A natural number is said to be a *base-g Niven number* if it is divisible by the sum of its base-g digits. In this talk, we show that, assuming Hooley's Riemann Hypothesis, the set of base-g Niven numbers is an additive basis, that is, there exists a positive integer C_g such that every natural number is the sum of at most C_g base-g Niven numbers. Then we pose some open problems of a similar flavor.

(81) **Wolfgang Schmid**, LAGA, University of Paris 8, France
Email: schmid@math.univ-paris13.fr
Title: Sequences of sets over finite abelian groups and weighted zero-sum sequences
Abstract: For a finite abelian group $(G, +)$, a sequence $g_1 \dots g_k$ over G is called a zero-sum sequence if $g_1 + \cdots + g_k = 0$ (we consider sequences that just differ by the ordering of the terms as equal). Given a set of weights Ω, for example integers or endomorphisms of the group, an Ω-weighted zero-sum sequence is a sequence $g_1 \dots g_k$ over G such that there is a choice

of weights $w_i \in \Omega$ such that $w_1 g_1 + \cdots + w_k g_k = 0$. The set of zero-sum sequences over G and the set of Ω-weighted zero-sum sequences form monoids with concatenation as operation.

We present some new results on the arithmetic of monoids of Ω-weighted zero-sum sequences, especially on their set of minimal distances. Moreover, we discuss a generalization of the concept of weighted zero-sum sequences. Instead of considering sequences of elements of the group, we study sequences of subsets of the group and call them zero-sum sequence if 0 is an element of the sum of the sets.

Joint work with S. Boukheche, K. Merito and O. Ordaz.

(82) **James Sellers**, University of Minnesota Duluth
Email: jsellers@d.umn.edu
Title: Sequentially congruent partitions and partitions into squares
Abstract: In recent work, Max Schneider and Robert Schneider studied a curious class of integer partitions called "sequentially congruent" partitions wherein the mth part is congruent to the $(m+1)$st part modulo m, and the smallest part is congruent to zero modulo the number of parts in the partition. After sharing a number of results that Schneider and Schneider proved regarding these sequentially congruent partitions, we will then prove that the number of sequentially congruent partitions of weight n is equal to the number of partitions of weight n where all parts are squares. We then show how our proof naturally generalizes to other classes of partitions where the parts are all kth powers for some fixed k. This is joint work with Robert Schneider and Ian Wagner.

(83) **Aliaksei Semchankau**, Steklov Mathematical Institute, Moscow
Email: aliaksei.semchankau@gmail.com
Title: A new bound for $A(A+A)$ for large sets
Abstract: Let G be a finite abelian group and let A, B be its 'dense' subsets. We show that popular values in $A+B$, and, more generally, any level-set of convolution $A * B$, can be approximated by a subset of G which we call *wrapper*.

Wrappers, which we introduce here, have some special structural features, making them suitable for both enumerative and additive-combinatorial purposes.

In particular, wrappers turn out to be useful when studying problems on sumsets, involving both set A and its inverse A^*. Assuming p to be a large prime number, we obtain:

- If set $A \subseteq \mathbb{F}_p^*$ is 'dense', then $|A + A^*| \geqslant (1 - o(1)) \min(2\sqrt{|A|p}, p)$.
- If sum-free set $A \subseteq \mathbb{F}_p^*$ satisfies $A = A^*$, then $|A| \leqslant p/9 + o(p)$.
- If set $A \subseteq \mathbb{F}_p^*, |A| = \alpha p$ satisfies $\alpha > 1/8$, then the set $\mathbb{F}_p^*$ is covered by $A(A+A)$.

(84) **Steve Senger**, Missouri State University
Email: stevensenger@gmail.com
Title: Upper and lower bounds on chains determined by angles
Abstract: We study a variant of the Erdős unit distance problem, concerning angles between successive triples of points chosen from a large finite point set. Specifically, given a large finite set of n points E, and a sequence of angles $(\alpha_1, \ldots, \alpha_k)$, we give upper and lower bounds on the maximum possible number of tuples of distinct points $(x_1, \ldots, x_{k+2}) \in E^{k+2}$ satisfying $\angle(x_j, x_{j+1}, x_{j+2}) = \alpha_j$ for every $1 \leq j \leq k$.

(85) **Oriol Serra** Universitat Politècnica de Catalunya, Barcelona
Email: oriol.serra@upc.edu
Title: Triangulations and the Brunn–Minkowski inequality
Abstract: Discrete versions of the Brunn–Minkowski inequality have been proposed by Ruzsa and by Gardner and Gronchi, which to some extent are not completely satisfactory. Matolcsi and Ruzsa conjectured a seemingly more appropriate discrete analogue of the Brunn–Minkowsky inequality. If $tr(A)$ denotes the number of triangles in a triangulation of the finite set A of points in the plane, the conjecture states that $tr(A+B)^{1/2} \geq tr(A)^{1/2} + tr(B)^{1/2}$. In the talk I will report on progress towards this conjecture and discuss possible extensions to general dimension d.

(86) **George Shakan**, University of Oxford
Email: george.shakan@gmail.com
Title: A large gap in a dilate of a set
Abstract: Let $A \subset \mathbb{F}_p$. How large of a gap can we find in some dilate of A? We discuss some progress and related questions.

(87) **I. D. Shkredov**, Steklov Mathematical Institute, Moscow
Email: ilya.Skredov@gmail.com
Title: On an application of higher energies to Sidon sets
Abstract: We show that for any finite set A and an arbitrary $\varepsilon > 0$ there is $k = k(\varepsilon)$ such that the higher energy $\mathsf{E}_k(A)$ is at most $|A|^{k+\varepsilon}$ unless A has a very specific structure. As an application we obtain that any finite subset A of the real numbers or the prime field either contains an additive Sidon–type subset of size $|A|^{1/2+c}$ or a multiplicative Sidon–type subset of size $|A|^{1/2+c}$.

(88) **Olivine Silier**, California Institute of Technology
Email: osilier@caltech.edu
Title: Structural Szemerédi-Trotter theorem for lattices.
Abstract: Incidence problems provide a framework for characterizing an underlying geometry and find applications beyond discrete geometry, spanning combinatorics, number theory, and harmonic analysis. A point and a line form an incidence if the point is on the line. When $|P| = |L| = n$, the Szemerédi-Trotter theorem states that the number of incidences between points from the set P and lines from the set L is $O(n^{4/3})$. The theorem is tight since there exist configurations with $\Theta(n^{4/3})$ incidences. Only two such configurations were known, one from Erdős, the other from Elekes.

In this work, we find a family of constructions (including these two) that spans all maximum-incidence constructions with a lattice of points. Moreover, while the Szemerédi-Trotter theorem has been known for nearly four decades, hardly anything is known about the structural problem: characterizing the configurations with $\Theta(n^{4/3})$ incidences. Here, we use an energy variant to derive a tight point-energy bound which depends on the geometry of the configuration. We also derive a variety of structural properties where the point set is a Cartesian product.
Joint work with Adam Sheffer.

(89) **Tom Slattery**, University of Warwick, UK
Email: T.Slattery@warwick.ac.uk
Title: On Fibonacci partitions
Abstract: The Fibonacci Partition function, OEIS A000119, counts the number of partitions of an integer into distinct Fibonacci numbers. This sequence made an appearance in the very first volume of the Fibonacci Quarterly in 1963 and, in general, behaves erratically. Sam Chow and I proved an exact formula for this function. I will present this proof as well as that of a similar formula for its mean value function, and determine the asymptotic behaviour.

(90) **Sophie Stevens**, Johan Radon Institute for Computational and Applied Mathematics (RICAM), Austria
Email: sophie.stevens@oeaw.ac.at
Title: On sumsets of convex functions
Abstract: The sum-product problem is a study of the extent to which additive and multiplicative structure cannot coexist. A close cousin of this is the question: To what extent does convexity destroy additive structure? In joint work with Audie Warren, we tackle this question by studying sums of convex functions in the reals. Specifically we prove that for any finite sets $A, B \subseteq \mathbb{R}$ and for any convex functions f and g, we have

$$|A+B|^{38}|f(A)+g(B)|^{38} \gtrsim |A|^{49}|B|^{49}\,.$$

This result can be used to obtain bounds on a number of two- and three-variable expanders of interest. I'll outline the context of our results and the techniques that enable them.

(91) **Tim Trudgian**, UNSW Canberra at ADFA
Email: t.trudgian@adfa.edu.au
Title: Twenty-four carats of Goldbach oscillations
Abstract: Addressing the Goldbach conjecture, like many additive problems, becomes easier when we look at averages. This approach goes back to Hardy and Littlewood. The main term in the average is known. I shall outline recent work (joint with Mike Mossinghoff, CCR, Princeton) showing that the lower order terms for 'Goldbach on the average' oscillate more than was known previously: these results are close to best possible.

(92) **Yuri Tschinkel**, New York University
Email: yuri.tschinkel@googlemail.com
Title: Arithmetic properties of equivariant birational types
Abstract: I will discuss arithmetic properties of new invariants in higher-dimensional equivariant geometry, introduced in joint work with Kontsevich–Pestun and Kresch.

(93) **Maciej Ulas**, Jagiellonian University, Krakow, Poland
Email: maciej.ulas@uj.edu.pl
Title: Equal values of certain partition functions via Diophantine equations
Abstract: Let $A \subset \mathbf{N}_+$ and let $P_A(n)$ denote the number of partitions of an integer n into parts from the set A. The aim of this paper is to prove several result concerning the existence of integer solutions of Diophantine equations of the form $P_A(x) = P_B(y)$, where A, B are certain finite sets.
Link to arXiv: `https://arxiv.org/abs/2102.05352`
Joint work with Szabolcs Tengely (University of Debrecen).

(94) **Geertrui Van de Voorde**, University of Canterbury, New Zealand
Email: geertrui.vandevoorde@canterbury.ac.nz
Title: On the product of elements with prescribed trace
Abstract: A question from finite geometry/combinatorics brought us to the following problem. Is it possible to write every element of a (finite) field as the product of two elements with prescribed trace? This problem is related to various other topics, such as PN-functions and polynomials with prescribed coefficients. In this talk, I will show how these problems are interrelated and how some techniques from number theory allowed us to answer this problem in particular cases, e.g. for finite fields of order q^n where $n \geq 5$.
Joint work with J. Sheekey and J.F. Voloch.

(95) **Robert Vaughan**, Pennsylvania State University
Email: rcv4@psu.edu
Title: On generating functions in additive number theory
Abstract: New asymptotic expressions for sums of the form

$$\sum_{n \leq P} e(\alpha_k n^k + \alpha_1 n)$$

are established with second and lower order terms beyond the classical main terms, and with improved error term estimates. As an application, it is shown that for almost all $\alpha_1 \in [0, 1)$, the bound

$$\sup_{\alpha_1 \in [0,1)} \Big| \sum_{1 \leq n \leq P} e(\alpha_1(n^3 + n) + \alpha_2 n^3) \Big| \ll P^{3/4+\varepsilon},$$

holds and that this is best possible. One surprising consequence is improved bounds for the fractal dimension of solutions to the Schrödinger and Airy equations.
Joint work with J. Brandes, S. T. Parsell, C. Poulias, G. Shakan.

(96) **Aled Walker**, Trinity College Cambridge, UK
Email: aw530@cam.ac.uk
Title: Effective results on the size and structure of sumsets
Abstract: Given a finite set $A \subset \mathbb{Z}^d$, in 1992 Khovanskii proved that there is some fixed polynomial P_A, with rational coefficients and degree at most d, such that $|NA| = P_A(N)$ for all sufficiently large N. But what does 'sufficiently large' mean in practice? Khovanskii's original proof was ineffective. Via other methods, effective bounds have been proved in a few special cases: when $d = 1$, due to Nathanson; when the convex hull of A is a d-simplex, due to Curran–Goldmakher; and when $|A| = d+2$, also due to Curran–Goldmakher. In this talk I will report on joint work with Andrew Granville and George Shakan, in which we prove an effective bound in the general setting. We will also discuss our related results on the structure of NA (for large N).

(97) **Audie Warren**, Johan Radon Institute for Computational and Applied Mathematics (RICAM), Austria
Email: audie.warren@oeaw.ac.at
Title: Additive and multiplicative Sidon sets
Abstract: A set of real numbers is called a Sidon set if there are no non-trivial solutions to the equation $a_1 + a_2 = a_3 + a_4$, with $a_i \in A$. Similarly, a *multiplicative* Sidon set is a set containing no non-trivial solutions to $a_1 a_2 = a_3 a_4$. In this talk we introduce a conjecture of Klurman and Pohoata, stating that every finite set $A \subseteq \mathbb{R}$ must contain a large additive or multiplicative Sidon set. In a joint work with Oliver Roche-Newton, we refute a strong form of this conjecture, giving a construction of a set of size N whose additive and multiplicative Sidon subsets are all of size $O(N^{2/3})$.

(98) **S. Kaylee Weatherspoon**, University of South Carolina
Email: SKW4@email.sc.edu
Title: A description of maximal non-biconnected unit distance graphs in the plane
Abstract: The Hadwiger-Nelson problem (chromatic number of the plane) asks for the least number of colors such that the entire plane in $\mathbb{R}^2$ can be colored without monochromatic unit distances. Beginning with a study of the chromatic number of disks in the plane, I'll present a description of maximal non-biconnected unit distance graphs in $\mathbb{R}^2$, along with diophantine questions associated with the development of these graphs.
Joint work with Joshua Cooper and Michael Filaseta.

(99) **James Wheeler**, University of Bristol, UK
Email: jw13032@bristol.ac.uk
Title: Incidence theorems for modular hyperbolae in positive characteristic
Abstract: In this talk we prove new incidence bounds between a plane point set, which is a Cartesian product, and a set of translates H of the hyperbola $xy = 1$, over a field of asymptotically large positive characteristic p. They improve recent bounds by Shkredov, which are based on using explicit incidence estimates in the early terminated procedure of repeated applications of the Cauchy-Schwarz inequality, underlying many qualitative results related to growth and expansion in groups. The improvement – both quantitative, plus we are able to deal with a general H, rather than a Cartesian product – is mostly due to a non-trivial "intermediate" bound on the number of k-rich Möbius hyperbolae in positive characteristic. In addition, we make an observation that a certain energy-type quantity in the context of H can be bounded via the L^2-moment of the Minkowski distance in H and can therefore fetch the corresponding estimates apropos of the Erdős distinct distance problem.

(100) **Trevor Dion Wooley**, Purdue University
Email: twooley@purdue.edu
Title: Rudin, polynomials, and nested efficient congruencing
Abstract: In 1960, Rudin made an influential conjecture concerning the number of squares to be found in an arithmetic progression of length N (there should be at most order square-root of N such squares). Obtaining estimates uniform in the modulus of the arithmetic progression is difficult, and experts will recognise that this is the point of Rudin's Conjecture. Recent work of Bourgain and Demeter obtains an upper bound in the much easier problem in which one seeks explicit dependence on the modulus of the arithmetic progression, a result anteceded by one of Uchiyama in 1976. We explore what these approaches have to say about the problem corresponding to that of Rudin in which the squares are replaced by values of a polynomial of higher degree. The underlying method, applying nested efficient congruencing, ought to be of utility elsewhere.

(101) **Max Wenqiang Xu**, Stanford University
Email: maxxu@stanford.edu
Title: Discrepancy in modular arithmetic progressions
Abstract Celebrated theorems of Roth and of Matoušek and Spencer together show that the discrepancy of arithmetic progressions in the first n positive integers is $\Theta(n^{1/4})$. We study the analogous problem in the $\mathbb{Z}_n$ setting. We asymptotically determine the logarithm of the discrepancy of arithmetic progressions in $\mathbf{Z}^n$ for all positive integer n. We further determine up to a constant factor the discrepancy of arithmetic progressions in $\mathbf{Z}^n$ for many n. For example, if $n = p^k$ is a prime power, then the discrepancy of arithmetic progressions in $\mathbf{Z}^n$ is $\Theta(n^{1/3+r_k/(6k)})$, where $r_k \in \{0, 1, 2\}$ is the remainder when k is divided by 3. This solves a problem of Hebbinghaus and Srivastav. Joint work with Jacob Fox and Yunkun Zhou.

(102) **Catherine Yan**, Texas A & M University
Email: huafei-yan@tamu.edu
Title: Vector parking functions with rational boundary
Abstract: Vector parking functions are sequences of non-negative integers whose order statistics are bounded by a given integer sequence $u = (u_0, u_1, u_2, ...)$. The classical parking function corresponds to the integer $u = (1, 2, 3, ...)$. In this talk we present a method to compute the exponential generating function when u is given by a linear function with rational slope. Our techniques include the theory of fractional power series, an analog of the Newton-Puiseux Theorem, and the theory of Goncharov polynomials.

(103) **Chi Hoi Yip**, University of British Columbia, Canada
Email: kyleyip@math.ubc.ca
Title: Gauss sums and the maximum cliques in generalized Paley graphs of square order
Abstract: Let $GP(q, d)$ be the d-Paley graph defined on the finite field $\mathbb{F}_q$. It is notoriously difficult to improve the trivial upper bound $\sqrt{q}$ on the clique number of $GP(q, d)$. In this talk, we will investigate the connection between Gauss sums over a finite field and maximum cliques of their corresponding generalized Paley graphs. In particular, we show that the trivial upper bound on the clique number of $GP(q, d)$ attains if and only if $d \mid (\sqrt{q} + 1)$, which strengthens the previous related results by Broere-Döman-Ridley and Schneider-Silva, as well as improves the trivial upper bound on the clique number of $GP(q, d)$ when $d \nmid (\sqrt{q} + 1)$.

(104) **Qinghai Zhong**, Universität Graz, Austria
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Title: On product-one sequences over subsets of groups
Abstract: Let G be a group and let $G_0 \subseteq G$ be a subset. A sequence over G_0 means a finite unordered sequence of terms from G_0 with repetition allowed. A product-one sequence is a sequence whose elements can be ordered such that their product equals the identity element of the group. We study algebraic and arithmetic properties of monoids of product-one sequences over finite subsets of G and over the whole group G, with a special emphasis on the finite and infinite dihedral groups.