CANT 2022: Zoom Conference

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Twentieth Annual Workshop on
Combinatorial and Additive Number Theory
CUNY Graduate Center
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Abstracts

(1) **Emma Bailey**, CUNY Graduate Center
Email: ebailey@gc.cuny.edu
Title: Large deviations of Selberg's central limit theorem
Abstract: Selberg's celebrated central limit theorem shows that $\log \zeta(1/2+it)$ at a typical point t at height T behaves like a complex, centered Gaussian random variable with variance $\log \log T$. This talk will present recent results showing that the Gaussian decay persists in the large deviation regime, at a level on the order of the variance, improving on the best known bounds in that range. Time permitting, we will also present various applications, including on the maximum of the zeta function in short intervals.

This work is joint with Louis-Pierre Arguin.

(2) **Benjamin Baily**, Williams College
Email: bmb2@williams.edu
Title: Large sets are sumsets
Abstract: Let $[n] := \{0, 1, 2, \ldots, n\}$. Intuitively, all large subsets of $[n]$ have additive structure, and Roth famously made this precise by finding constants c, $N > 0$ such that for $n \geq N$, any subset of $[n]$ containing more than $\frac{cn}{\log \log n}$ elements must contain an arithmetic progression of length 3. We establish a different interpretation of the intuition by finding explicit constants $\alpha = \frac{1}{\log 2}$ and $\beta = \frac{1}{\log 1.325}$ such that, for sufficiently large n, we have:

(i) any subset of $[n]$ with more than $n - \alpha \log n$ elements has a nontrivial decomposition as the sum of two sets, and
(ii) there exists a subset of $[n]$ of size $n - \beta \log n$ at least that has no such decomposition.

We also prove, using these methods, a higher-dimensional analogue of results (i) and (ii). Notably, our threshold at which structure appears is far higher than Roth's.

This work was joint with Justine Dell, Sophia Dever, Adam Dionne, Henry Fleischmann, Leo Goldmakher, Gal Gross, Faye Jackson, Steven J. Miller, Ethan Pesikoff, Huy Tuan Pham, Luke Reifenberg, and Vidya Venkatesh.

(3) **Gautami Bhowmik**, Université de Lille, France
Email: gautami.bhowmik@univ-lille.fr
Title: Siegel zeros under Goldbach conjectures
Abstract: A Landau-Siegel zero is a possible though unwelcome counterexample to the Generalised Riemann Hypothesis. Proving its absence unconditionally is clearly a difficult problem. We will discuss some results by assuming plausible conjectures on the Goldbach problem: the Hardy-Litllewood one (1923), a weak form due to Fei (2016), and a weaker form that we studied more recently (Bhowmik-Halupczok, in: Proceedings of CANT 2019 and 2020). Continuing on these lines, Friedlander-Goldston-Iwaniec-Suriajaya (2022) showed that the assumption of Fei's conjecture is enough to disprove the existence of Siegel zeros.

(4) **Jörg Brüdern, Universität Göttingen, Germany**
Email: joerg.bruedern@mathematik.uni-goettingen.de
Title: Bracketed ternary additive problems
Abstract: The ternary additive problems of Waring's type (that is, sums of three potentially unlike powers) have attracted many workers in the additive theory of numbers. In this talk, we discuss several variants that involve brackets (that is, the integer part of certain monomials).

(5) **Yin Choi Cheng**, CUNY Graduate Center
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Title: Order type of shifts of morphic words
Abstract: The shifts of an infinite word $W = a_0a_1\cdots$ are the words $W_i = a_ia_{i+1}\cdots$. As a measure of the complexity of a word W, we consider the order-type of the set of shifts, ordered lexicographically. We will look at the order-type of shifts of morphic words over a finite alphabet that are not ultimately periodic. As a concrete example, we give the explicit ordering among shifts of the Thue-Morse word. The order type of shifts of the Fibonacci word will be discussed. We then give special consideration to uniform morphisms on 3 letters.

(6) **Jin-Hui Fang**, Nanjing University of Information Science and Technology, China
Email: fangjinhui1114@163.com
Title: Representation functions avoiding integers with density zero
Abstract: For a nonempty set A of integers and any integer n, denote $r_A(n)$ by the number of representations of n of the form $n = a + a'$, where $a \leq a'$ and $a, a' \in A$ and $d_A(n)$ by the number of pairs (a, a') with $a, a' \in A$ such that $n = a - a'$. In 2008, Nathanson considered the representation function with infinitely many zeros. Following Nathanson's work, we proved that, for any set T of integers with density zero, there exists a sequence A of integers such that $r_A(n) = 1$ for all integers $n \notin T$ and $r_A(n) = 0$ for all integers $n \in T$, and $d_A(n) = 1$ for all positive integers n. We will also present our recent results on representation functions.

(7) **Leonid Fel**, Technion – Israel Institute of Technology, Israel
Email: lfel@cv.technion.ac.il
Title: Commutative monoid of self-dual symmetric polynomials
Abstract: We consider a set $\mathfrak{RS}(\lambda, S_n)$ of self- and skew-reciprocal polynomials in λ, of degree mn, where $m \in \mathbb{Z}_{\geq}$, $n \in \mathbb{Z}_{>}$, based on polynomial invariants $I_{n,r}(\mathbf{x}^n)$ of symmetric group S_n, acting on the Euclidean space $\mathbb{E}^n$ over the field of real numbers $\mathbb{R}$, where $\mathbf{x}^\mathbf{n} = \{x_1, \ldots, x_n\} \in \mathbb{E}^n$. We prove that $\mathfrak{RS}(\lambda, S_n)$ exhibits a commutative monoid under multiplication. Real solutions $\lambda(\mathbf{x}^\mathbf{n})$ of skew-reciprocal equations have many remarkable properties: a homogeneity of the 1st order, a duality under inversion of variables $x_i \to x_i^{-1}$ and function $\lambda \to \lambda^{-1}$, a monotony of $\lambda(\mathbf{x}^\mathbf{n})$ with respect to every x_i and others. We find the bounds of $\lambda(\mathbf{x}^\mathbf{n})$ which are given by arithmetic and harmonic means of the set $\{x_1, \ldots, x_n\}$.

(8) **Henry Fleischmann**, University of Michigan, and
Ethan Pesikoff, Yale University
Email: henryfl@umich.edu and ethan.pesikoff@yale.edu
Title: Angle variants of the Erdős distinct distance problem
Abstract: The Erdős distinct distance problem is a ubiquitous problem in discrete geometry. Less well known is Erdős' distinct angle problem, the problem of finding the minimum number of distinct angles between n non-collinear points in the plane. We provide new upper and lower bounds on a broad class of distinct angle problems. We show that the number of distinct angles formed by n points in general position is $O(n^{\log_2(7)})$, providing the first non-trivial bound for this quantity. We introduce several new asymptotically optimal configurations. We also show, via a probabilistic argument, that the minimum size of a maximal subset of n points in general position admitting only unique angles is $\Omega(n^{1/5})$ and $O(n^{\log_2(7)/3})$. These are the best known bounds on this quantity.

This work is joint with Hongyi Hu, Faye Jackson, Steven J. Miller, Evyindur Palsson, and Charles Wolf.

(9) **Mikhail Gabdullin**, Steklov Mathematical Institute, Moscow, Russia
Email: gabdullin.mikhail@yandex.ru
Title: A conjecture of Cilleruelo and Cordoba and divisors in a short interval
Abstract: Let $E(A) = \#\{(a_1, a_2, a_3, a_4) \in A^4 : a_1 + a_2 = a_3 + a_4\}$ denote the additive energy of a set $A \subset \mathbf{N}$, and let $\mathbb{T} = \mathbf{R}/\mathbf{Z}$ and $\|f\|_4 = \left(\int_{\mathbb{T}} |f(t)|^4 dt\right)^{1/4}$. It is well-known that

$$E(\{n^2 : n \leq N\}) = \left\| \sum_{n \leq N} e^{2\pi i n^2 x} \right\|_4^4 \asymp N^2 \log N,$$

while we trivially have $E(A) \geq |A|^2$. In 1992, J. Cilleruelo and A. Cordoba proved that $E(\{n^2 : N \leq n \leq N + N^\gamma\}) \asymp N^{2\gamma}$ for any $\gamma \in (0, 1)$, and conjectured a much more general bound (again, for any $\gamma \in (0,1)$)

$$\left\| \sum_{N \leq n \leq N+N^\gamma} a_n e^{2\pi i n^2 x} \right\|_4 \leq C(\gamma) \left(\sum_{N \leq n \leq N+N^\gamma} |a_n|^2 \right)^{1/2}.$$

While this bound is easy to prove for $\gamma \leq 1/2$, it seems to be open for any $\gamma > 1/2$. We prove this for all $\gamma < \frac{\sqrt{5}-1}{2} = 0.618...$ and present a connection between this problem and a conjecture of I. Ruzsa: for any $\epsilon > 0$ there exists $C(\epsilon) > 0$ such that any positive integer N has at most $C(\epsilon)$ divisors in the interval $[N^{1/2}, N^{1/2} + N^{1/2-\epsilon}]$.

(10) **Krystian Gajdzica**, Jagiellonian University, Kraków, Poland
Email: krystian.gajdzica@im.uj.edu.pl
Title: Some inequalities for the multicolor restricted partition function $p_{\mathcal{A}}(n,k)$
Abstract: For a non-decreasing sequence of positive integers $\mathcal{A} = (a_i)_{i=1}^\infty$ and a fixed integer $k \geqslant 1$, the multicolor restricted partition function $p_{\mathcal{A}}(n,k)$ counts the number of partitions of n with parts in the multiset $\{a_1, a_2, \ldots, a_k\}$. The talk is devoted to some multiplicative inequalities related to $p_{\mathcal{A}}(n,k)$. Among other things, we will examine: the Bessenrodt-Ono inequality for $p_{\mathcal{A}}(n,k)$, the log-concavity of the sequence $(p_{\mathcal{A}}(n,k))_{n=1}^\infty$, the higher order Turán property and other similar phenomena.

(11) **Filip Gawron**, Jagiellonian University, Poland
Email: filipux.gawron@student.uj.edu.pl
Title: Sign behavior of sums of weighted numbers of partitions
Abstract: Let A be a subset of the positive integers. By an A-partition of n we understand the representation of n as a sum of elements from the set A. For given i, $n \in \mathbb{N}$, by $c_A(i,n)$ we denote the number of A-partitions of n with exactly i parts. In the talk I will describe several results concerning the sign behaviour of the sequence $S_{A,k}(n) = \sum_{i=0}^n (-1)^i i^k c_A(i,n)$, for fixed $k \in \mathbb{N}$. I will focus on the periodicity of the sequence of signs for different forms of A. Finally, I will also mention some conjectures and questions that arose naturally during our research.

The talk is based on a joint work with Maciej Ulas (Jagiellonian University).

(12) **Rachel Greenfeld**, UCLA
Email: greenfeld@ucla.edu
Title: Translational tilings
Abstract: Translational tiling is a covering of a space using translated copies of some building blocks, called the tiles, without any positive measure overlaps. Which are the possible ways that a space can be tiled? In the talk, we will discuss the study of this question as well as its applications, and report on recent progress, joint with Terence Tao.

(13) **Li Guo**, Rutgers University - Newark
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Title: Renormalization of quasisymmetric functions
Abstract: The algebra of quasisymmetric functions (QSym) has played a central role in multiple zeta values and a large class of combinatorial algebraic structures related to symmetric functions. A natural linear basis of QSym is the set of monomial quasisymmetric functions defined by compositions, that is, vectors of positive integers. Extending such a definition for weak compositions, that is, vectors of nonnegative integers, leads to divergent expressions. This phenomenon is closely related to the divergency of multiple zeta values with nonpositive integer arguments.

We apply the method of renormalization in the spirit of Connes and Kreimer to address the divergency, and realize weak composition quasisymmetric functions as power series. The resulting Hopf algebra has the Hopf algebra of quasisymmetric functions as both a Hopf subalgebra and a Hopf quotient algebra.

This is joint work with Houyi Yu and Bin Zhang.

(14) **Shruti Hegde**, Ramakrishna Mission Vivekananda Educational and Research Institute, India
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Title: Weighted zero-sum constants and inverse results
Abstract: A weighted generalization of classical zero-sum constants was introduced by Adhikari *et al.* in 2006 and has been an active area of research since then. In the last fifteen years, weighted zero-sum constants for $\mathbb{Z}_n$ with several interesting weight sets have been found. In this talk, we take up the problem of determining the exact values and providing bounds of the weighted Davenport constant of $\mathbb{Z}_n$ with some new weight sets.

Next, we consider a weighted generalization of the *the Erdős-Ginzburg-Ziv constant*. Let G be a finite abelian group with $\exp(G) = n$. For a positive integer k and a non-empty subset A of $[1, n-1]$, the arithmetical invariant $\mathsf{s}_{kn,A}(G)$ is defined to be the least positive integer t such that any sequence S of t elements in G has an A-*weighted zero-sum subsequence* of length kn. We give the exact value of $\mathsf{s}_{kq,A}(G)$, for integers $k \geq 2$ and $A = \{1, 2\}$, where G is an abelian p-group with $rank(G) \leq 4$, p is an odd prime and $exp(G) = q$. Our method consists of a modification of a polynomial method of Rónyai.

Lastly, we consider the questions regarding inverse problems for the weighted zero-sum constants of $\mathbb{Z}_n$. An inverse problem is the problem of characterizing all the weighted *zero-sum free sequences* over $\mathbb{Z}_n$ of specific lengths for the particular weight sets under consideration.

This work was joint with Sukumar Das Adhikari and partly with Md Ibrahim Molla and Subha Sarkar.

(15) **Norbert Hegyvari**, Eötvös Loránd University and Alfréd Rényi Institute of Mathematics, Hungary
Email: hegyvari@renyi.hu
Title: Boolean functions defined on pseudo-recursive sequences
Abstract: We define Boolean functions on hypergraphs with edges having big intersections, and an opposite situation, hypergraphs which are thinly intersecting induced by pseudo-recursive sequences. A sequence X is said to be *pseudo-recursive sequence* if the identity $x_{n+1} = m_{n+1}x_n + b_{j_{n+1}}$ holds, where $b_{j_{n+1}} \in \{b_1, b_2, \dots b_k\})$ for $n \geq 0$. As a main result, we estimate of their supports. The tools come from additive combinatorics and the uncertainty inequality.

(16) **Russell Jay Hendel**, Towson University
Email: RHendel@Towson.Edu
Title: A system of four simultaneous recursions
Abstract: This paper further generalizes a recent result of Shannon and Ollerton who resurrected an old identity due to Ledin. This paper generalizes the Ledin-Shannon-Ollerton result to all metallic sequences. The results give closed formulas for the sum of products of powers of the first n integers with the first n members of the metallic sequence. Three key innovations of this paper are (i) reducing the proof of the generalization to the solution of a system of 4 simultaneous recursions; (ii) skillful use of the shift operation to prove equality of polynomials; and (iii) new OEIS sequences arising from the coefficients of the four polynomial families satisfying the four simultaneous recursions.

(17) **Brad Isaacson**, New York City College of Technology (CUNY)
Email: bisaacson@citytech.cuny.edu
Title: On a polynomial reciprocity theorem of Carlitz
Abstract: Carlitz proved a powerful reciprocity theorem for generalized Dedekind-Rademacher sums. Among its many consequences was an interesting polynomial reciprocity theorem which holds under a certain restriction of its parameters. Carlitz remarked that it was unclear how this restriction could be removed. In this talk, we remove this restriction and obtain a generalization of Carlitz's polynomial reciprocity theorem.

(18) **Faye Jackson**, University of Michigan, and
Luke Reifenberg, University of Notre Dame
Email: alephnil@umich.edu and lreifenb@nd.edu
Title: The Generalized Bergman game
Abstract: P. Baird-Smith A. Epstein, K. Flint, and S. J. Miler (2018) created the *Zeckendorf Game*, a two-player game which takes as an input a positive integer n and, using moves related to the Fibonacci recurrence relation, outputs the unique decomposition of n into a sum of non-consecutive Fibonacci numbers. Following this work and that of G. Bergman (1957), which proved the existence and uniqueness of such φ-decompositions, we formulate the *Bergman Game* which outputs the unique decomposition of n into a sum of non-consecutive powers of φ, the golden mean.

We then formulate *Generalized Bergman Games*, which use moves based on an arbitrary non-increasing positive linear recurrence relation and output the unique decomposition of n into a sum of non-adjacent powers of β, where β is the dominating root of the characteristic polynomial of the chosen recurrence relation. We prove that the longest possible Generalized Bergman game on an initial state S with n summands terminates in $\Theta(n^2)$ time, and we also prove that the shortest possible Generalized Bergman game on an initial state terminates between $\Omega(n)$ and $O(n^2)$ time. We also show a linear bound on the maximum length of the tuple used throughout the game.

This is joint work with Benjamin Baily, Justine Dell, Irfan Durmic, Henry Fleischmann, Isaac Mijares, Steven J. Miller, Ethan Pesikoff, Alicia Smith Reina, and Yingzi Yang.

(19) **Renling Jin**, College of Charleston
Email: jinr@cofc.edu
Title: Hyper-hyper-hyper-integers and a simple proof of Szemerédi's theorem
Abstract: In a conference five years ago, T. Tao reported his effort to simplify Szemerédi's original combinatorial proof of Szemerédi's theorem using nonstandard analysis. We continued his effort and presented a simple proof of the theorem for $k = 4$ in CANT 2020. In this talk, we will present a simple proof of the theorem for all k. One of the main simplifications is that a Tower of Hanoi type induction used by Szemerédi as well as Tao is replaced by a straightforward induction. In the proof the integers with three levels of infinities are used.

(20) **Gergely Kiss**, Alfréd Rényi Institute of Mathematics, Hungary
Email: kigergo57@gmail.com
Title: Fuglede's conjecture on the direct product of finite abelian groups
Abstract: We investigate Fuglede's conjecture on the direct product of abelian groups and its connection to the conjecture on $\mathbb{R}^n$ for $n \geq 2$. We overview the earlier results: Some important constructions will be shown, which disproves the conjecture in higher dimensions, and some techniques and ideas will be presented, which serves to prove the conjecture for certain abelian groups. Finally we will discuss some developments of the most recent directions of research. This talk is closely related to Gábor Somlai's talk about Fuglede's conjecture on the cyclic group and one dimensional cases, and to the talk of Thomas Fallon who will present the detailed proof of Fuglede's conjecture for $\mathbb{Z}_p^2 \times \mathbb{Z}_q^2$.

(21) **Jakub Konieczny**, Claude Bernard University Lyon 1, France
Email: jakub.konieczny@gmail.com
Title: Automatic semigroups
Abstract: Automatic sequences, that is, sequences computable by finite automata, have been extensively studied from a variety of perspectives, including combinatorics, number theory, dynamics and theoretical computer science. Classification problems are a natural class of questions in

the theory of automatic sequences. In particular, the problem of classifying automatic multiplicative sequences has attracted considerable attention, culminating in complete classification which we obtained in joint work with Clemens Müllner and Mariusz Lemańczyk. The subject of my talk will be an extension of this line of inquiry, which we pursue in joint work with Oleksiy Klurman. Under mild technical assumptions, we classify all automatic multiplicative semigroups, that is, all sets E of integers which are closed under multiplication and such that the indicator function 1_E is automatic. Additionally, we show (again, under mild technical assumptions) that if E, F are automatic sets with $E \cdot F \subset E$ then E must contain a large essentially periodic component. This leads to potentially interesting open problems concerning products of automatic sets.

(22) **Noah Kravitz**, Princeton University
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Title: Zero patterns of derivatives of polynomials
Abstract: Motivated by recent work of Nathanson, we study the zero patterns of derivatives of polynomials. For P a polynomial of degree n and $\Lambda = (\lambda_1, \ldots, \lambda_m)$ an m-tuple of distinct complex numbers, we consider the $m \times (n+1)$ *dope matrix* $D_P(\Lambda)$ whose ij-entry equals 1 if $P^{(j)}(\lambda_i) = 0$ and equals 0 otherwise (for $1 \le i \le m$, $0 \le j \le n$). We address several natural questions: When m is 1 or 2, what do the possible dope matrices look like, and how many are there? What can we say about general upper bounds on the number of $m \times (n+1)$ dope matrices? For which m-tuples Λ is the number of $m \times (n+1)$ dope matrices maximized? Does every $\{0, 1\}$-matrix appear as the left-most portion of some dope matrix?

Based on joint work with Noga Alon and Kevin O'Bryant.

(23) **Thái Hoàng Lê**, University of Mississippi
Email: leth@olemiss.edu
Title: Bohr sets in sumsets in countable abelian groups
Abstract: A *Bohr set* in an abelian topological group G is a subset of the form

$$B(K, \epsilon) = \{g \in G : |\chi(g) - 1| < \epsilon \, \forall \chi \in K\}$$

where K is a finite subset of the dual group $\widehat{G}$. A classical theorem of Bogolyubov says that if $A \subset \mathbf{Z}$ has positive upper density δ, then $A+A-A-A$ contains a Bohr set $B(K, \epsilon)$ where $|K|$ and ϵ depend only on δ. While the same statement for $A - A$ is not true (a result of Kříž), Bergelson and Ruzsa proved that if $r + s + t = 0$, then $rA + sA + tA$ contains a Bohr set (here $rA = \{ra : a \in A\}$). We investigate this phenomenon in more general groups G, where rA, sA, tA are replaced by images of A under certain endomomorphisms of G. It is also natural to ask for partition analogues of the Bergelson-Ruzsa theorem. In CANT 2021, I discussed our results in compact abelian groups (generalizations of $\mathbf{R}/\mathbf{Z}$). In this talk, I will discuss our progress on countable discrete abelian groups (generalizations of $\mathbf{Z}$). The key ingredients are certain transference principles which allow us to transfer the results from compact groups to discrete countable groups.

This talk is joint work with Anh Le, and with Anh Le and John Griesmer.

(24) **Paolo Leonetti**, Università "Luigi Bocconi", Milano, Italy
Email: leonettipaolo@gmail.com
Title: The G.C.D. of n and the nth Fibonacci number
Abstract: Let $(F_n)_{n\geq 1}$ be the sequence of Fibonacci numbers, defined as usual by $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for all positive integers n; and let $\mathcal{A}$ be the set of all integers of the form $\gcd(n, F_n)$, for some positive integer n. In this talk we shall illustrate the following result on $\mathcal{A}$.
Theorem. *For all $x \geq 2$, we have*

$$\#\mathcal{A}(x) \gg \frac{x}{\log x}.$$

On the other hand, $\mathcal{A}$ has zero asymptotic density. The proofs rely on a result of Cubre and Rouse (PAMS, 2014) which gives, for each positive integer n, an explicit formula for the density of primes p such that n divides the rank of appearance of p, that is, the smallest positive integer k such that p divides F_k.

(25) **Jared Duker Lichtman**, University of Oxford
Email: jared.lichtman@maths.ox.ac.uk
Title: A proof of the Erdős primitive set conjecture
Abstract: A set of integers greater than 1 is primitive if no member in the set divides another. Erdős proved in 1935 that the series of $1/(n \log n)$, ranging over n in A, is uniformly bounded over all choices of primitive sets A. In 1988 he asked if this bound is attained for the set of prime numbers. In this talk we describe recent work which answers Erdős' conjecture in the affirmative. We will also discuss applications to old questions of Erdős, Sárközy, and Szemerédi from the 1960s.

(26) **Ariane Masuda**, New York City College of Technology, CUNY
Email: amasuda@citytech.cuny.edu
Title: Rédei permutations with the same cycle structure
Abstract: Permutation polynomials over finite fields have been extensively studied over the past decades. Among the major challenges in this area are the questions concerning their cycle structures as they capture relevant properties, both theoretically and practically. In this talk we focus on a family of permutation polynomials, the so called Rédei permutations. Although their cycle structures are known, there are other related questions that can be investigated. For example, when do two Rédei permutations have the same cycle structure? We give a characterization of such pairs, and present explicit families of Rédei permutations with the same cycle structure. We also discuss some results regarding Rédei permutations with a particularly simple cycle structure, consisting of 1- and j-cycles only, when j is 4 or a prime number. The case $j = 2$ is specially important in some applications. We completely describe Rédei involutions with a prescribed cycle structure, and show that the only Rédei permutations with a unique cycle structure are the involutions. This is joint work with Juliane

Capaverde and Virgínia Rodrigues.

(27) **Piotr Miska**, Jagiellonian University, Kraków, Poland
Email: piotr.miska@uj.edu.pl
Title: On (non-)realizibility of Stirling numbers
Abstract: We say that a sequence $(a_n)_{n\in\mathbb{N}_+}$ of non-negative integers is realizable if there exists a set X and a mapping $T : X \to X$ such that a_n is the number of fixed points of T^n. For each $k \in \mathbb{N}_+$ and $j \in \{1, 2\}$ we define a sequence $S_k^{(j)} = (S^{(j)}(n + k - 1, k))_{n\in\mathbb{N}_+}$, where $S^{(j)}(n, k)$ is the Stirling number of the j-th kind (in case of $j = 1$ we consider unsigned Stirling numbers). The aim of the talk is to prove that $S_k^{(2)}$ is realizable if and only if $k \in \{1, 2\}$, while for $k \geq 3$ the sequence $S_k^{(2)}$ is almost realizable with a failure $(k-1)!$, i. e. $(k-1)!S_k^{(2)}$ is realizable. Moreover, I will show that for each $k \in \mathbb{N}_+$ the sequence $S_k^{(1)}$ is not almost realizable, i. e. for any $r \in \mathbb{N}_+$ the sequence $rS_k^{(1)}$ is not realizable.

The talk is based on a joint work with Tom Ward (Newcastle, UK).

(28) **Mel Nathanson**, Lehman College (CUNY)
Email: melvyn.nathanson@lehman.cuny.edu
Title: Multiplicity interpolation of polynomials
Abstract: Interpolation problems related to the theorems of Descartes, Budan-Fourier, and Sturm in the theory of equations.

(29) **Péter Pál Pach**, TU Budapest, Hungary
Email: p.p.pach@gmail.com
Title: Colouring the smooth numbers
Abstract: For a given n, can we colour the positive integers using precisely n colours in such a way that for any a, the numbers $a, 2a, \dots, na$ all get different colours? This question is still open in general. I will present a survey of known results and some other problems it leads to.

This is joint work with Andr0s Caicedo and Thomas Chartier.

(30) **Huy Pham**, Stanford University,
Email: huypham@stanford.edu
Title: Homogeneous structures in subset sums and applications
Abstract: In recent joint works with David Conlon and Jacob Fox, we develop novel techniques which allow us to prove a diverse range of results relating to subset sums. In the one-dimensional case, our techniques imply the existence of long *homogeneous arithmetic progressions* in the set of subset sums under a variety of assumptions. This allows us to resolve a number of longstanding open problems, including: solutions to the three problems of Burr and Erdős on Ramsey complete sequences, for which Erdős later offered a combined total of \$350; analogous results for the new notion of density complete sequences; the solution to a conjecture of Alon and Erdős on the minimum number of colors needed to color the positive integers less than n so that n cannot be written as a monochromatic sum; the exact determination of an extremal function introduced by Erdős and Graham

on sets of integers avoiding a given subset sum; and, answering a question reiterated by several authors, a homogeneous strengthening of a result of Szemerédi and Vu on long arithmetic progressions in subset sums. In follow-up work in the multi-dimensional case, we show the existence of large *homogeneous generalized arithmetic progressions* in the set of subset sums of sufficiently large subsets of $[n]$, yielding a strengthening of a seminal result of Szemerédi and Vu. As an application, we make progress on the Erdős–Straus non-averaging sets problem, showing that every subset A of $[n]$ of size at least $n^{\sqrt{2}-1+o(1)}$ contains an element which is the average of two or more other elements of A. This gives the first polynomial improvement on a result of Erdős and Sárközy from 1990.

(31) **Paul Pollack**, University of Georgia
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Title: Weak uniform distribution of certain arithmetic functions
Abstract: For any fixed integer q, it is a classical result (implicit in work of Landau, and perhaps known earlier) that Euler's function $\phi(n)$ is a multiple of q asymptotically 100% of the time. Thus, $\phi(n)$ is very far from being uniformly distributed mod q in the usual sense (unless $q = 1$!). On the other hand, Narkiewicz has proved that $\phi(n)$ is weakly uniformly distributed mod q whenever q is coprime to 6; weakly means that every coprime residue class mod q gets its fair share of values $\phi(n)$, from among the n with $\phi(n)$ coprime to q. In fact, Narkiewicz proves this not just for ϕ but for a wide class of polynomially-defined multiplicative functions. In this talk, we will consider these weak uniform distribution problems with an eye towards obtaining wide ranges of uniformity in the modulus q.

This is joint work with Noah Lebowitz-Lockard and Akash Singha Roy.

(32) **Sean Prendiville**, Lancaster University, UK
Email: s.prendiville@lancaster.ac.uk
Title: Adapting the circle method for colourings
Abstract: Fix your favourite Diophantine equation. If each integer is coloured red, blue or green, how many solutions to your equation have all variables the same colour? We discuss how to adapt the Hardy-Littlewood circle method to yield a lower bound in certain problems of this flavour.

(33) **Alex Rice**, Millsaps College
Email: riceaj@millsaps.edu
Title: New results in classical and arithmetic Ramsey theory
Abstract: For $r, k \in \mathbf{N}$, Ramsey's Theorem says that there exists a least positive integer $R_r(k)$ such that every r-coloring of the edges of a complete graph on $N \geq R_r(k)$ vertices yields a monochromatic complete subgraph on k vertices. This fact can be applied to deduce Schur's Theorem, which says that there exists a least positive integer $S_r(k)$ such that every r-coloring of $\{1, 2, \ldots, N\}$ for $N \geq S_r(k)$ yields a monochromatic solution to the equation $x_1 + x_2 + \cdots + x_{k-1} = x_k$. Here we discuss new findings related to these two classical results. First, we derive explicit upper bounds on $R_r(k)$, established through the pigeonhole principle and careful bookkeeping, that

improve upon previously documented bounds. Second, we present an extension of Schur's Theorem to higher-dimensional integer lattices, with the additional restriction that the vectors on the left hand side of the equation are linearly independent.

This includes joint work with six (at the time) Millsaps College undergraduate students: Vishal Balaji, Powers Lamb, Andrew Lott, Dhruv Patel, Sakshi Singh, and Christine Rose Ward.

(34) **Sinai Robins**, University of Sao Paulo, Brazil
Email: srobins@ime.usp.br
Title: The covariogram and an extension of Siegel's formula
Abstract: We extend a formula of Carl Ludwig Siegel in the geometry of numbers. Siegel's original formula assumed that there is exactly one lattice point in the interior of the body, while here we relax that condition, so that the body may contain an arbitrary number of interior lattice points. Our extension involves a lattice sum of the covariogram for any compact set $\mathcal{K} \subset \mathbb{R}^d$, where the covariogram of $\mathcal{K}$ at $x \in \mathbb{R}^d$ is defined by $\mathrm{vol}(\mathcal{K} \cap (\mathcal{K} + x))$. The proof hinges on a variation of the Poisson summation formula which we derive here, and the Fourier methods herein also allow for more general admissible sets. One of the consequences of these results is a new characterization of multi-tilings of Euclidean space by translations, using the lower bound on lattice sums of such covariograms. The classical result known as Van der Corput's inequality, also follows immediately from the main result, as well as a new spectral formula for the volume of a compact set.

This is joint work with Michel Faleiros Martins.

(35) **Anurag Sahay**, University of Rochester
Email: anuragsahay@rochester.edu
Title: Moments of the Hurwitz zeta function with rational shifts
Abstract: The Hurwitz zeta function is a shifted integer analogue of the Riemann zeta function, for shift parameters $0 < \alpha \leqslant 1$. We consider the moments of the Hurwitz zeta function on the critical line $\Re s = 1/2$ for rational shifts $\alpha = a/q$. In this case, the Hurwitz zeta function decomposes as a linear combination of Dirichlet L-functions, which leads us into investigating moments of products of L-functions.

If time permits, we will briefly discuss these moments for irrational shift parameters α, which shall dovetail into Trevor Wooley's talk on our joint work with Winston Heap.

(36) **Carlo Sanna**, Politecnico di Torino, Italy
Email: carlo.sanna@polito.it
Title: Membership in random ratio sets
Abstract: Let $\mathcal{A}$ be a random set constructed by picking independently each element of $\{1, \ldots, n\}$ with probability $\alpha \in (0, 1)$. Several authors studied combinatorial/number-theoretic objects involving $\mathcal{A}$, including the sum set $\mathcal{A} + \mathcal{A}$, the product set $\mathcal{A}\mathcal{A}$, and the ratio set $\mathcal{A}/\mathcal{A}$. Generalizing a previous result of Cilleruelo and Guijarro-Ordóñez, we give a formula for the

probability that a rational number q belongs to the ratio set $\mathcal{A}/\mathcal{A}$. Moreover, we give some results about formulas for the probability of the event $\bigvee_{i=1}^{k}(q_i \in \mathcal{A}/\mathcal{A})$, where $q_1, \ldots, q_k$ are rational numbers, showing that they are related to the study of the connected components of certain graphs. Finally, we provide some open question for future research.

(37) **James Sellers**, University of Minnesota Duluth
Email: jsellers@d.umn.edu
Title: Relating the crank of a partition and smallest missing parts
Abstract: The primary goal of this talk is to demonstrate a natural connection between the smallest missing part of an integer partition (commonly referred to as the "mex" of the partition) and the concept of the crank of a partition. After providing a brief history of the crank of a partition a la Dyson as well as Andrews and Garvan, we will utilize straightforward generating function manipulations to make this connection. We will then consider additional results on the mex statistic based on parity, and we will also demonstrate connections between the crank and Frobenius symbols which satisfy certain conditions.

This work is joint with Brian Hopkins, Dennis Stanton, and Ae Ja Yee.

(38) **Steven Senger**, Missouri State University
Email: stevensenger@gmail.com
Title: Distinct dot products, convexity, and $AA + 1$
Abstract: We discuss recent developments in estimating the number of distinct dot products determined by a large finite set of n points in the plane. The improvement comes from improved understanding of the multiplicative structure of an additively shifted product set, $AA + 1$, when A is a large finite subset of the real numbers. This breakthrough was made possible by new additive combinatorial results about convex sets of numbers.

(39) **Bartosz Sobolewski**, Jagiellonian University, Kraków, Poland
Email: bartosz.sobolewski@uj.edu.pl
Title: Monochromatic arithmetic progressions in binary words associated with pattern sequences
Abstract: Let $e_v(n)$ denote the number of occurrences of a pattern v in the binary expansion of $n \in \mathbb{N}$. In the talk we consider monochromatic arithmetic progressions in the class of words $(e_v(n) \bmod 2)_{n\geq 0}$ over $\{0, 1\}$, which includes the Thue–Morse word $\mathbf{t}$ $(v = 1)$ and a variant of the Rudin–Shapiro word $\mathbf{r}$ $(v = 11)$. So far, the problem of exhibiting long progressions and finding an upper bound on their length has mostly been studied for $\mathbf{t}$ and certain generalizations. We show that analogous results hold for $\mathbf{r}$. In particular, we prove that a monochromatic arithmetic progression of difference $d \geq 3$ starting at 0 in $\mathbf{r}$ has length at most $(d + 3)/2$, with equality infinitely often. We also compute the maximal length of progressions of differences $2^k - 1$ and $2^k + 1$. Some weaker results for a general pattern v are provided as well.

(40) **Gábor Somlai**, Eötvös Loránd University and Alfréd Rényi Institute of Mathematics, Hungary
Email: zsomlai@gmail.com
Title: Fuglede's conjecture, the one dimensional case
Abstract: Fuglede conjectured that a bounded measurable set (in $\mathbb{R}^n$) is spectral if and only if it is a tile. The conjecture was also confirmed by Fuglede for sets whose tiling complement is lattice and for spectral sets one of whose spectrums is a lattice. The conjecture was disproved by Tao by constructing a spectral set in $\mathbb{Z}_3^5$, which is not a tile and lifted it to the 5 dimensional Euclidean space.

The conjecture is open only in dimensions 1 and 2. The 1 dimensional case is directly connected with the one of finite cyclic groups and to the so called Coven-Meyerowitz conjecture. One of the main aims of the talk is to present some of the methods developed that lead to our recent results.

(41) **Johann Thiel**, New York City College of Technology (CUNY)
Email: jthiel@citytech.cuny.edu
Title: Solving the membership problem for certain subgroups of $SL_2(\mathbb{Z})$
Abstract: For positive integers u and v, let $L_u = \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix}$ and $R_v = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix}$. Let $G_{u,v}$ be the group generated by L_u and R_v. The membership problem for $G_{u,v}$ asks the following question: Given a 2-by-2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, is there a relatively straightforward method for determining if M is a member of $G_{u,v}$? In the case where $u = 2$ and $v = 2$, Sanov was able to show that simply checking some divisibility conditions for a, b, c and d is enough to make this determination. We answered this question in the case where $u, v \geq 3$ by finding a characterization of matrices M in $G_{u,v}$ in terms of the short continued fraction representation of $\frac{b}{d}$, extending some results of Esbelin and Gutan. By modifying our previous work, we are able to further extend our previous result to the more difficult case where $u, v \geq 2$ with $uv \neq 4$.

This is joint work with Sandie Han, Ariane M. Masuda, and Satyanand Singh.

(42) **Ognian Trifonov**, University of South Carolina
Email: trifonov@math.sc.edu
Title: Lattice points close to ovals, arcs, and helixes
Abstract: In 1972 Schinzel showed that the largest distance between three lattice points on a circle of radius R is at least $\sqrt[3]{2}R^{1/3}$. We generalize Schinzel's result to ovals and arcs with bounded curvature in the plane and lattice points close to the curve. Furthermore, we extend the result to the case of affine lattices. Finally, we obtain similar results when the curve is a helix in three dimensional space.

(43) **Tim Trudgian**, UNSW Canberra at the Australian Defence Force Academy
Email: t.trudgian@adfa.edu.au
Title: Dont believe the Fake Mus!
Abstract: Perhaps your favourite sum is biased ... leaning a little towards the negative, perhaps? Perhaps your sum is suspiciously similar to the Moebius function $\mu(n)$? What can we do with such fake mus? Come along to find out, and together, we can make arithmetic great again!

This is joint work with Greg Martin (UBC) and Mike Mossinghoff (CCR, Princeton).

(44) **Maciej Ulas**, Jagiellonian University, Kraków, Poland
Email: maciej.ulas@gmail.com
Title: Solutions of certain meta-Fibonacci recurrences
Abstract: We investigate the solutions of certain meta-Fibonacci recurrences of the form $f(n) = f(n - f(n-1)) + f(n-2)$ for various sets of initial conditions. In the case when $f(n) = 1$ for $n \leq 1$, we prove that the resulting integer sequence is closely related to the function counting binary partitions of a certain type (independently of the value of $f(2) \in \mathbb{N}$).

The talk is based on a joint work with Bartosz Sobolewski.

(45) **Ethan Patrick White**, University of British Columbia
Email: epwhite@math.ubc.ca
Title: Erdős' minimum overlap problem
Abstract: In 1955 Erdős posed the following problem. Let n be a positive integer and $A, B \subset [2n]$ be a partition of $[2n]$ such that $|A| = |B| = n$. For any such partition and integer $-2n < k < 2n$, define M_k to be the number of solutions $(a, b) \in A \times B$ to $a - b = k$. Estimate the size of the function

$$M(n) = \min_{A \cup B = [2n]} \max_{-2n<k<2n} M_k,$$

where the minimum is taken over all partitions of $[2n]$ into equal-sized sets. Many upper and lower estimates were obtained over the following decades, and the state of the art is $0.356 < M(n)/n < 0.381$. We use elementary Fourier analysis to translate the problem to a convex optimization program and obtain the new lower bound $M(n)/n > 0.379$.

(46) **Trevor D. Wooley**, Purdue University
Email: twooley@purdue.edu
Title: Shifted analogues of the divisor function
Abstract: Suppose that θ is irrational. Then almost all elements $\nu \in \mathbb{Z}[\theta]$ that may be written as a k-fold product of the shifted integers $n + \theta$ ($n \in \mathbb{N}$) are thus represented essentially uniquely. We discuss this and related paucity problems.

Most of this work is joint with Winston Heap and Anurag Sahay.

(47) **Max Wenqiang Xu**, Stanford University
Email: maxxu@stanford.edu
Title: On a Turán conjecture and random multiplicative functions
Abstract: We show that if f is the random completely multiplicative function, the probability that $\sum_{n\leq x}\frac{f(n)}{n}$ is positive for every x is at least $1-10^{-40}$. For large x we prove an asymptotic upper bound of $O(\exp(-\exp(\frac{\log x}{C\log\log x})))$ on the probability that a particular truncation is negative. This is joint work with Rodrigo Angelo.

(48) **Ajmain Yamin**, CUNY Graduate Center
Email: ayamin@gradcenter.cuny.edu
Title: The exceptional automorphism of S_6 explained with colored maps
Abstract: Among all symmetric groups, S_6 is the only one with a nontrivial outer automorphism, In this talk, I will describe a new way to understand the exotic embedding of $S_5 \hookrightarrow S_6$ in terms of 5-colored complete regular maps on the torus. This provides a visual explanation for the existence of the exceptional automorphism of S_6.

(49) **Catherine Yan**, Texas A&M University
Email: huafei-yan@tamu.edu
Title: Multivariate Gončarov polynomials and integer sequences
Abstract: Univariate delta Gončarov polynomials arise when the classical Gončarov interpolation problem in numerical analysis is modified by replacing derivatives with delta operators. When the delta operator under consideration is the backward difference operator, we acquire the univariate difference Gončarov polynomials, which have a combinatorial relation to lattice paths in the plane with a given right boundary. In this talk, we extend several algebraic and analytic properties of univariate Gončarov polynomials to the multivariate case with both the derivative and backward difference operators. We then establish a combinatorial interpretation of multivariate Gončarov polynomials in terms of certain constraints on d-tuples of integer sequences. This motivates a connection between multivariate Gončarov polynomials and a higher-dimensional generalized parking function, the **U**-parking function, from which we derive several enumerative results based on the theory of delta operators.

This talk is based on joint work with Ayo Adeniran and Lauren Snider.

(50) **Daodao Yang**, Graz University of Technology, Austria
Email: yang@tugraz.at
Title: Extreme values of derivatives of the Riemann zeta function, log-type GCD sums, and spectral norms
Abstract: First I will recall the research on greatest common divisor (GCD) sums and extreme values of the Riemann zeta function. The motivation for the study and the connection between the two problems will be discussed. Then I will explain how to establish lower bounds for maximums of $|\zeta^{(\ell)}(\sigma+it)|$ when $\sigma\in[\frac{1}{2},1]$, $\ell\in\mathbf{N}$. One of my results states that as $T\to\infty$, uniformly for all positive integers $\ell\leqslant(\log_3 T)/(\log_4 T)$, we have

$$\max_{T\leqslant t\leqslant 2T}\left|\zeta^{(\ell)}\Big(1+it\Big)\right|\geqslant\big(\mathbf{Y}_\ell+o(1)\big)\left(\log_2 T\right)^{\ell+1},$$

where $\mathbf{Y}_\ell = \int_0^\infty u^\ell \rho(u)du$, and $\rho(u)$ denotes the Dickman function. This generalizes results of Bohr-Landau and Littlewood on $|\zeta(1+it)|$ in 1910s. The tools are Soundararajan's resonance methods and ingredients are certain combinatorial optimization problems. On the other hand, assuming the Riemann hypothesis, we have $|\zeta^{(\ell)}(1+it)| \ll_\ell (\log\log t)^{\ell+1}$. Then I will talk on the log-type GCD sums $\Gamma_\sigma^{(\ell)}(N)$, which I define it as follows

$$\Gamma_\sigma^{(\ell)}(N) := \sup_{|\mathcal{M}|=N} \frac{1}{N} \sum_{m,n\in\mathcal{M}} \frac{(m,n)^\sigma}{[m,n]^\sigma} \log^\ell\left(\frac{m}{(m,n)}\right) \log^\ell\left(\frac{n}{(m,n)}\right),$$

where the supremum is taken over all subsets $\mathcal{M} \subset \mathbb{N}$ with size N. I will explain how $\Gamma_\sigma^{(\ell)}(N)$ can be related to $|\zeta^{(\ell)}(1+it)|$ and how to prove that $(\log\log N)^{2+2\ell} \ll_\ell \Gamma_1^{(\ell)}(N) \ll_\ell (\log\log N)^{2+2\ell}$, which generalizes Gál's theorem (corresponding to the case $\ell = 0$). The lower bounds could be used to produce large values of $|\zeta^{(\ell)}(1+it)|$. Using a random model for the zeta function via methods of Lewko-Radziwiłł, upper bounds for spectral norms on α-line are established, when $\alpha \to 1^-$ with certain fast rates. As a corollary, upper bounds of correct order of the log-type GCD sums are established.

(51) **Wijit Yangjit**, University of Michigan
Email: yangjit@umich.edu
Title: On the Montgomery–Vaughan weighted generalization of Hilbert's inequality
Abstract: Hilbert's inequality states that

$$\left| \sum_{m=1}^{N} \sum_{\substack{n=1 \\ n\neq m}}^{N} \frac{z_m \overline{z_n}}{m-n} \right| \le C_0 \sum_{n=1}^{N} |z_n|^2,$$

where C_0 is an absolute constant. In 1911, Schur showed that the optimal value of C_0 is π.

In 1974, Montgomery and Vaughan proved a weighted generalization of Hilbert's inequality and used it to estimate mean values of Dirichlet series. This generalized Hilbert inequality is important in the theory of the large sieve. The optimal constant C in this inequality is known to satisfy $\pi \le C < \pi + 1$. It is widely conjectured that $C = \pi$. In this talk, I will describe the known approaches to obtain an upper bound for C, which proceed via a special case of a parametric family of inequalities. We analyze the optimal constants in this family of inequalities. A corollary is that the method in its current form cannot imply an upper bound for C below 3.19.

(52) **Chi Hoi Yip**, University of British Columbia
Email: kyleyip@math.ubc.ca
Title: Asymptotics for the number of directions determined by $[n] \times [n]$ in $\mathbb{F}_p^2$
Abstract: Let p be a prime and n a positive integer such that $\sqrt{\frac{p}{2}}+1 \leq n \leq \sqrt{p}$. For any arithmetic progression A of length n in $\mathbb{F}_p$, we establish an asymptotic formula for the number of directions determined by $A \times A \subset \mathbb{F}_p^2$. The key idea is to reduce the problem to counting the number of solutions to the bilinear Diophantine equation $ad+bc=p$ in variables $1 \leq a,b,c,d \leq n$; our asymptotic formula for the number of solutions is of independent interest.

Joint work with Greg Martin and Ethan White.

(53) **Qinghai Zhong**, University of Graz, Austria
Email: qinghai.zhong@uni-graz.at
Title: On monoids of weighted zero-sum sequences
Abstract: Let G be an additive finite abelian group and $\Gamma \subset \mathrm{End}(G)$ be a subset of the endomorphism group of G. A sequence $S = g_1 \cdot \ldots \cdot g_\ell$ over G is a (Γ-)weighted zero-sum sequence if there are $\gamma_1, \ldots, \gamma_\ell \in \Gamma$ such that $\gamma_1(g_1) + \ldots + \gamma_\ell(g_\ell) = 0$. We study algebraic and arithmetic properties of monoids of weighted zero-sum sequences.