CANT 2023

.

Twenty-first Annual Workshop on Combinatorial and Additive Number Theory

CUNY Graduate Center

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Abstracts

(1) **Ruben Ascoli**, Princeton University
Email: rascoli@princeton.edu
Andrew W. Keisling, University of Michigan
Email: keislina@umich.edu
Title: Sum and difference sets in generalized dihedral groups
Abstract: Given a group G, we say that a set $A \subseteq G$ has more sums than differences (MSTD) if $|A+A| > |A-A|$, has more differences than sums (MDTS) if $|A+A| < |A-A|$, or is sum-difference balanced if $|A+A| = |A-A|$. Recently, Miller and Vissuet studied the frequency of these types of subsets in arbitrary finite groups G and proved that almost all subsets $A \subseteq G$ are sum-difference balanced as $|G| \to \infty$. For the dihedral group D_{2n}, they conjectured that of the remaining sets, most are MSTD.

We extend the conjecture to generalized dihedral groups $D = \mathbb{Z}_2 \ltimes G$, where G is an abelian group of size n and the nonidentity element of $\mathbb{Z}_2$ acts by inversion. We make further progress on the conjecture by considering subsets with a fixed number of rotations and reflections. By bounding the expected number of overlapping sums, we show that the collection $\mathcal{S}_{D,m}$ of subsets of the generalized dihedral group D of size m has more MSTD sets than MDTS sets when $6 \leq m \leq c_j\sqrt{n}$ for $c_j = 1.3229/\sqrt{111+5j}$, where j is the number of elements in G with order at most 2. We also analyze the expectation for $|A+A|$ and $|A-A|$ for $A \subseteq D_{2n}$, providing an explicit formula for $|A-A|$ when n is prime.

Joint work with Justin Cheigh, Guilherme Zeus Dantas e Moura, Ryan Jeong, Astrid Lilly, Steven J. Miller, Prakod Ngamlamai, Matthew Phang.

(2) **Emma Bailey**, CUNY Graduate Center
Email: ebailey@gc.cuny.edu
Title: Atypical behaviour of the Riemann zeta function
Abstract: Selberg's celebrated central limit theorem shows that $\log\zeta(1/2+\mathrm{it})$ at a typical point t at height T behaves like a complex, centered Gaussian random variable with variance $\log\log T$. In this talk I will present some theoretical and numerical results regarding behavior of $\zeta(1/2+\mathrm{it})$ outside of the range where the central limit theorem is known to hold.

(3) **Jonathan Chapman**, University of Bristol, UK
Email: jonathan.chapman@bristol.ac.uk
Title: Ramsey properties of polynomial equations
Abstract: A system of Diophantine equations is called partition regular if it admits monochromatic solutions with respect to any finite colouring of the positive integers. In the 1930s, Rado obtained necessary and sufficient conditions for arbitrary systems of linear equations to be partition regular. However, it is only with recent breakthroughs in analytic number theory and additive combinatorics that we have been able to obtain an analogous characterisation of partition regularity for large families of polynomial equations. In this talk, I will report on these recent developments.

Joint work with Sam Chow (University of Warwick).

(4) **Robert Donley**, Queensborough Community College (CUNY)
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Title: Path enumeration for antimagic squares
Abstract: Students typically encounter path counting on rectangular posets as an application of binomial coefficients. If one recasts this model in terms of order-raising operators, the Jordan canonical form gives an alternative description of Clebsch-Gordan coefficients without representation theory. At the same time, the underlying parameter space consists of semimagic squares of size three, and path counting methods there also yield such coefficients in terms of certain hypergeometric series. Semimagic squares of larger size present formidable difficulties, but antimagic squares, as defined by R. Stanley, retain basic features from the case of size three. We present a model of antimagic squares for path counting and identify two families of zeros for the corresponding hypergeometric series.

This talk reports on work from the Summer 2022 RAMMP REU at City College (CUNY). Joint work with Arnav Krishna (Pierrepont School) and Zixuan Ye (University of Buffalo).

(5) **Jin-Hui Fang**, School of Mathematical Sciences, Nanjing Normal University, Nanjing 210023, PR China
Email: fangjinhui1114@163.com
Title: On the density of bounded bases
Abstract: For a nonempty set A of integers and an integer n, let $r_A(n)$ be the number of representations of n in the form $n = a + a'$, where $a \leq a'$ and $a, a' \in A$, and $d_A(n)$ be the number of representations of n in the form $n = a - a'$, where $a, a' \in A$. The binary support of a positive integer n is defined as the subset $S(n)$ of nonnegative integers consisting of the exponents in the binary expansion of n, i.e., $n = \sum_{i \in S(n)} 2^i$, $S(-n) = -S(n)$ and $S(0) = \emptyset$. For real number x, let $A(-x, x)$ be the number of elements $a \in A$ with $-x \leq a \leq x$. The famous Erdős-Turán Conjecture states that if A is a set of positive integers such that $r_A(n) \geq 1$ for all sufficiently large n, then $\limsup_{n\to\infty} r_A(n) = \infty$. In 2004, Nešetřil and Serra initially introduced the notation of "bounded" property and confirmed the Erdős-Turán conjecture for a class of *bounded* bases. They also proved that, there exists a set A of integers satisfying $r_A(n) = 1$ for all integers n and

$|S(x)\bigcup S(y)| \le 4|S(x+y)|$ for $x, y \in A$. On the other hand, Nathanson proved that there exists a set A of integers such that $r_A(n) = 1$ for all integers n and $2\log x/\log 5 + c_1 \le A(-x,x) \le 2\log x/\log 3 + c_2$ for all $x \ge 1$, where c_1, c_2 are absolute constants. Following these results, we prove that, there exists a set A of integers such that: $r_A(n) = 1$ for all integers n and $d_A(n) = 1$ for all positive integers n, $|S(x)\bigcup S(y)| \le 4|S(x+y)|$ for $x, y \in A$ and $A(-x,x) > (4/\log 5)\log\log x + c$ for all $x \ge 1$, where c is an absolute constant. Furthermore, we also construct a family of arbitrarily spare such sets A.

(6) **Leonid Fel**, Technion – Israel Institute of Technology, Israel
Email: lfel@cv.technion.ac.il
Title: Symmetric (not complete intersection) numerical semigroups and syzygy identities
Abstract: We consider symmetric (not complete intersection) numerical semigroups $\langle d_1, \ldots, d_m\rangle$ of arbitrary embedding dimension m, minimally generated by a set of m positive integers, such that $\gcd(d_1, \ldots, d_m) = 1$. We derive identities for degrees of syzygies of such semigroups and find the lower bound for their Frobenius numbers that generalizes recent results for $m = 4, 5, 6$.

(7) **Mikhail Gabdullin**, Steklov Mathematical Institute, Moscow, Russia
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Title: Prime avoiding numbers is a basis of order two
Abstract: For a positive integer n, let $F(n)$ be the distance from n to the nearest prime number. In 2015 Ford, Heath-Brown, and Konyagin introduced the notion of *prime avoidance*: a number n is called a prime avoiding number with constant c, if

$$F(n) \geqslant c\frac{\log n \log\log n \log\log\log\log n}{(\log\log\log n)^2}.$$

They proved that for any positive integer k there exist $c = c(k) > 0$ and infinitely many k-th powers which are prime avoiding with constant c. Later Maier and Rassias extended this result to k-th prime powers and a stronger prime avoidance.

We use the method from the recent breakthrough work of Ford, Konyagin, Maynard, Pomerance, and Tao on consecutive composite values of polynomials to prove the following theorem:

Every sufficiently large positive integer N can be represented as $N = n_1 + n_2$, where

$$F(n_i) \geqslant (\log N)(\log\log N)^{1/325565}, \quad i = 1, 2.$$

This is joint work with Artyom Radomskii (HSE University).

(8) **Krystian Gajdzica**, Jagiellonian University, Krakw, Poland
Email: krystian.gajdzica@im.uj.edu.pl
Title: Beyond the log-concavity of the restricted partition function
Abstract: For a fixed parameter $k \in \mathbb{N}_+$ and a sequence $\mathcal{A} = (a_i)_{i=1}^{\infty}$ of positive integers, the restricted partition function $p_{\mathcal{A}}(n,k)$ enumerates

those partitions of n whose parts belong to the multiset $\{a_1, a_2, \ldots, a_k\}$. The main issues of our interests will be both the r-log-concavity problem and the higher order Turán inequalities for $p_{\mathcal{A}}(n,k)$. However, we will also discuss some basic generalizations of the log-concavity which correspond to the following inequalities:

$$p_{\mathcal{A}}^2(n,k) > p_{\mathcal{A}}(n-m,k)p_{\mathcal{A}}(n+m,k),$$
$$p_{\mathcal{A}}^2(an,k) > p_{\mathcal{A}}(an-bn,k)p_{\mathcal{A}}(an+bn,k),$$
$$(n^2-1)^{\alpha}p_{\mathcal{A}}^2(n,k) > n^{2\alpha}p_{\mathcal{A}}(n-1,k)p_{\mathcal{A}}(n+1,k),$$

where $a, b, m \in \mathbb{N}_+$ and $\alpha \in \mathbb{R}_+$ are fixed such that $a > b$. For each of the problems, the efficient solution will be presented.

(9) **Zhenchao Ge**, ShanghaiTech University, China
Email: gezc@shanghaitech.edu.cn, gezhenchao@gmail.com
Title: A Weyl-type inequality for irreducible elements in function fields, with applications
Abstract: We establish a Weyl-type estimate for exponential sums over irreducible elements in function fields. As an application, we generalize an equidistribution theorem of Rhin. Our estimate works for polynomials with degree higher than the characteristic of the field, a barrier to the traditional Weyl differencing method. In this talk, we briefly introduce Lê-Liu-Wooley's original argument for ordinary Weyl sums (taken over all elements), and how we generalize it to estimate bilinear exponential sums with general coefficients.

This is joint work with Thái Hoàng Lê (Mississippi) and Yu-Ru Liu (Waterloo).

(10) **Robert Groth**, University of South Carolina
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Title: Constructing generalized Sierpínski numbers
Abstract: A Sierpínski number is a positive odd integer k such that $k \cdot 2^n + 1$ is composite for all $n \in \mathbb{Z}^+$. Fix an integer A with $2 \leq A$. We show there exists a positive odd integer k such that $k \cdot a^n + 1$ is composite for all integers $a \in [2, A]$ and all $n \in \mathbb{Z}^+$.

This is joint work with Michael Filaseta and Thomas Luckner.

(11) **C. Sinan Gunturk**, Courant Institute, NYU
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Title: Density of the Bernstein lattices
Abstract: Let $\mathcal{L}(\mathcal{P}_n)$ be the lattice of polynomials of degree at most n with integer coefficients. We define the Bernstein lattice $\mathcal{L}(\mathcal{B}_n)$ to be the set of all polynomials with integer coefficients with respect to the Bernstein basis

$$p_{n,k}(x) := \binom{n}{k} x^k (1-x)^{n-k},\ k = 0, \ldots, n.$$

$\mathcal{L}(\mathcal{B}_n)$ is a coarse sublattice of $\mathcal{L}(\mathcal{P}_n)$, of index $\prod_{k=0}^{n} \binom{n}{k} \sim \exp(cn^2)$.

Let $\mathcal{C}_{\mathbb{Z}}([0,1])$ consist of all real-valued continuous functions on $[0,1]$ that take on integer values at 0 and 1. It is well-known, due to Pál, Kakeya, and

Chlodovsky, that $\bigcup_n \mathcal{L}(\mathcal{P}_n)$ is dense in $\mathcal{C}_{\mathbb{Z}}([0,1])$ with respect to the uniform metric. We extend this result and show that in fact $\bigcup_n \mathcal{L}(\mathcal{B}_n)$ is dense in $\mathcal{C}_{\mathbb{Z}}([0,1])$ as well. Our result comes with an approximation algorithm and an effective bound on the error of approximation.

Time permitting, the talk will also include some applications of this result to neural networks.

Joint work with Weilin Li (City College, CUNY).

(12) **Norbert Hegyvari**, Rényi Institute, Budapest, Hungary
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Title: Covering shrinking polynomials by progressions
Abstract: Erdős introduced the quantity $S = T\sum_{i=1}^{T} X_i$, where $X_1, \ldots, X_T$ are arithmetic progressions, and cover the square numbers up to N. He conjectured that S is close to N, i.e. the square numbers cannot be covered "economically" by arithmetic progressions. Sárközy confirmed this conjecture and proved that $S \geq cN/\log^2 N$. In the talk, we extend this to shrinking polynomials and so-called $\{X_i\}$ quasi-progressions.

(13) **Harald Helfgott**, Universität Göttingen, Germany and CNRS, France
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Title: Explicit bounds on sums of the Möbius functions
Abstract: Let $M(x)$ be the Mertens function $M(x) = \sum_{n\leq x}\mu(n)$. Most of us are used to thinking of the problem of estimating $M(x)$ as being essentially equivalent to the problem of estimating the number of primes up to x (i.e., the Prime Number Theorem). If we want explicit bounds, however, the problem of bounding $M(x)$ becomes by far the harder of the two problems. The main reason is that, while the residue of $-\zeta'(s)/\zeta(s)$ at a zero of $\zeta(s)$ is just the order of the zero, we do not have a good way to control the residues of $1/\zeta(s)$. (There are bounds for $1/\zeta(s)$ inside the zero-free region, but their constants are very large.)

Up to now, the best bounds on $M(x)$ have been either (a) based on elementary methods that amount to delicate refinements of Chebyshev-Mertens, or (b) based on iterative processes that combine (a) with explicit versions of PNT. (The best results of these two kinds to date are due to Daval (unpublished) and Ramaré, respectively.) The same is true of sums of the form $m(x) = \sum_{n\leq x}\mu(n)/n$, $\check{m}(x) = \sum_{n\leq x}\mu(n)\log(x/n)/n$, etc., which appear often in analytic number theory.

We give considerably stronger bounds on $M(x)$, $m(x)$ and $\check{m}(x)$ by means of an analytic approach based on mean-value theorems.

Joint work with Andrés Chirre.

(14) **Russell Jay Hendel**, Towson University
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Title: Recursions, closed forms, and characteristic polynomials of the circuit array
Abstract: One modern graph metric represents an electrical circuit with a graph whose edges are replaced with resistors and the so-called resistance

distance between the nodes is determined by calculating the electrical resistance in the circuit. Electrical circuit theory provides functions that allow "reduction" of one circuit to another circuit where the resistance distance between certain vertices are preserved. Recently there has been study of a graph, representable in the Cartesian plan as an n-grid, n rows of upright equilateral triangles, all of whose edges are labeled one. It is possible to reduce the n-grid to an $(n-1)$-grid with resistance preserving operations. The collections of successive reductions has many interesting properties. In this talk we continue to study a "slice" of this collection of grids represented by the Circuit Array, an infinite array of rational functions. We show that certain closed forms, recursions, and characteristic polynomials (annihilators) emerge. One surprising result is that the annihilators of the numerators and denominators of the underlying rational functions exclusively have roots which are integral powers of 9.

Joint work with Emily J. Evans.

(15) **Robert Hough**, Stony Brook University
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Title: Subconvexity of Sato-Shintani zeta functions
Abstract: Selberg's Eisenstein series in n complex variables, with a symmetric group of functional equations, is just one example of a huge family (containing the standard L-functions, conjecturally containing the Selberg class) of zeta functions constructed by Sato and Shintani. I will describe an approach, joint with Eun Hye Lee, for proving subconvexity in this family. This work 'won the game', which I can try to explain, also.

(16) **Alex Iosevich**, University of Rochester
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Title: Point configurations in vector spaces over finite fields
Abstract: We are going to consider several variants of the Erdős-Falconer distance problem in vector spaces over finite fields and emphasize the connection between discrete and continuous methods.

(17) **Brad Isaacson**, NYC College of Technology (CUNY)
Email: brad.isaacson71@citytech.cuny.edu
Title: On some polynomial reciprocity formulas
Abstract: Carlitz proved a powerful reciprocity theorem for generalized Dedekind-Rademacher sums. Among its many consequences was an interesting polynomial reciprocity theorem which holds under a certain restriction of its parameters. Carlitz remarked that it was unclear how this restriction could be removed. In last year's CANT talk, we removed this restriction and obtained a generalization of Carlitz's polynomial reciprocity theorem. In this talk, we generalize it further. As a corollary, we get the polynomial reciprocity theorem of Beck and Kohl.

(18) **Ryan Jeong**, University of Pennsylvania
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Title: Progress On the relative sizes of complements of generalized sumsets
Abstract: Given a subset A of $\{1, 2, \ldots, N\}$, its sum set and difference set are given by

$$A + A \ = \ \{a_1 + a_2 : a_1, a_2 \in A\}, \qquad A - A \ = \ \{a_1 - a_2 : a_1, a_2 \in A\}.$$

A problem of recent interest has been to understand the relative sizes of the sum and difference sets. We might expect that usually $|A - A| > |A + A|$ as addition commutes, but Martin and O'Bryant proved that a positive proportion of the 2^N subsets A of $\{1, 2, \ldots, N\}$ are sum-dominated, while Hegarty and Miller establish a threshold phenomenon if we independently include elements of $\{1, 2, \ldots, N\}$ in A with probability $p(N)$: $\frac{|A-A|}{|A+A|} \xrightarrow[N\to\infty]{p} 2$ if $p(N) = o(N^{-1/2})$, this ratio decreases almost surely to 1 as c increases to ∞ for $p(N) = cN^{-1/2}$, and $\frac{|(A+A)^c|}{|(A-A)^c|} \xrightarrow[N\to\infty]{p} 2$ if $N^{-1/2} = o(p(N))$.

More recently, Hogan and Miller studied the relative sizes of generalized sum and difference sets of the form $A_{s,d} = A + A + \cdots + A - A - A - \cdots - A$, with s sums and d differences, $s + d = h$ and $p(N) = cN^{-\delta}$ for $\delta \geq (h-1)/h$. They generalized the results of Hegarty and Miller by establishing that $\frac{|A_{s_1,d_1}|}{|A_{s_2,d_2}|} \xrightarrow[N\to\infty]{p} \frac{s_2!d_2!}{s_1!d_1!}$ if $\delta > \frac{h-1}{h}$ and this ratio decreases almost surely to 1 as c increases to ∞ for $p(N) = cN^{-\frac{h-1}{h}}$. The behavior in the "slow decay regime $\delta < (h-1)/h$ was left open. We make progress in this direction by showing that, with probability $1 - o(1)$,

$$\frac{|A^c_{h,0}|}{|A^c_{h-d,d}|} \ \leq \ d + 1 + o(1),$$

which, in particular, establishes that the slow decay results of Hegarty and Miller do not generalize like the other two regimes for all choices of s and d.

Joint work with Steven J. Miller.

(19) **Mizan Khan**, Eastern Connecticut State University
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Title: Interior hulls and continued fractions
Abstract: The interior hull of a lattice polygon is the convex closure of the lattice points in the interior of the polygon. We will give a concrete description of the interior hull of any lattice parallelogram.

We begin by focusing on clean parallelograms. A clean parallelogram in $\mathbb{R}^2$ is a lattice parallelogram whose boundary contains no lattice points other than its vertices. Using unimodular maps we can identify a clean parallelogram with a parallelogram whose vertices are $(0, 0), (1, 0), (a, n)$ and $(a + 1, n)$ with $0 < a < n$ and $\gcd(a, n) = 1$. Following Stark's geometric approach to continued fractions we show that the convergents of the continued fraction of n/a (viewed as lattice points) appear in a one-to-two correspondence with the vertices of the interior hull of this parallelogram. Once we have given a concrete description of the interior hull of any clean parallelogram, we can easily adapt our work to give a concrete description

of the interior hull of an arbitrary lattice parallelogram.
This is joint work with Gabriel Khan, Riaz Khan and Peng Zhao.

(20) **Gergely Kiss**, Alfréd Rényi Institute of Mathematics, Budapest, Hungary
Email: kigergo57@gmail.com
Title: Recent progress on Fuglede's conjecture on $\mathbb{Z}_p^3$
Abstract: In my talk I present the recent developments in Fuglede's conjecture on $\mathbb{Z}_p^3$. This problem is particularly interesting, since it is known that the conjecture holds on $\mathbb{Z}_p$ and on $\mathbb{Z}_p^2$, and does not hold on $\mathbb{Z}_p^d$ for $d \geq 4$. Although the problem on $\mathbb{Z}_p^3$ is still open in general, I show a new concept, which leads to some combinatorial and finite geometric problems by using the concept of weak tilings and discrete Fourier analysis. My talk is based on joint work with D. Matolcsi, M. Matolcsi and G. Somlai and I also discuss the results of R. D. Malikiosis.

(21) **Sándor Kiss**, Budapest University of Technology and Economics, Hungary
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Title: Problems and results on additive representation functions associated to linear forms
Abstract: Let $k \geq 2$ and $\lambda_1, \dots, \lambda_k$ be fixed positive integers. For a set A of nonnegative integers, the additive representation function associated to linear forms is

$$R_{A,\underline{\lambda}}(n) = |\{(a_1, \dots, a_k) \in A^k : \lambda_1 a_1 + \ldots + \lambda_k a_k = n\}|.$$

In my talk I would like to summarize our recent results about representation functions associated to linear forms. We will extend an earlier result of Nathanson to representation functions associated for linear forms. Furthermore, we will describe all the k-tuples $\underline{\lambda} = (\lambda_1, \dots, \lambda_k)$ and the sets of nonnegative integers A with $R_{A,\underline{\lambda}}(n) = 1$ for every nonnegative integer n. We also have several open problems for further research.
This is joint work with Csaba Sándor.

(22) **Sergei Konyagin**
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Title: Large gaps between sums of two squarefull numbers
Abstract: A positive integer n is called squarefull or powerful if in its factorization $n = p_1^{\alpha_1} \dots p_r^{\alpha_r}$ we have $\alpha_i \geq 2$ for all i. We consider that 0 is also a squarefull number. Thus, a number is squarefull if and only if it can be represented as $n = a^2 b^3$ for some $a, b \in \mathbb{Z}_+$.

Let W be the set of all sums of two squarefull numbers. Blomer (2005) proved that

$$W(x) := |W \cap [1, x]| = \frac{x}{(\log x)^{\alpha + o(1)}} \quad (x \to \infty),$$

where $\alpha = 1 - 2^{-1/3} = 0.20\ldots$.

As suggested by Shparlinski, we study large gaps between elements of W. Namely, for $x > 1$ define $M(x)$ as the length of the largest subinterval of $[1, x]$ without elements of W. Blomer's result implies that $M(x) \geq$

$(\log x)^{\alpha+o(1)}$ as $x \to \infty$ since the largest gap is at least as large as the average gap. We improve this estimate.

Theorem: For $x \geq 3$ we have $M(x) \geq c(\log x)/(\log\log x)^2$ where $c > 0$ is an absolute constant.

Joint work with Alexander Kalmynin.

(23) **Tomasz Kowalczyk**, Jagiellonian University in Kraków, Poland
Email: tomek.kowalczyk@uj.edu.pl
Title: On Waring numbers of henselian rings II
Abstract: Let $n > 1$ be a positive integer and R be a commutative ring with unity. We define the nth Waring number $w_n(R)$ as the smallest positive integer g such that every sum of nth powers of elements of R can be written as a sum of at most g nth powers of elements of R.

Let R be a henselian local ring with residue field k of nth level $s_n(k)$. We give some upper and lower bounds for the nth Waring number $w_n(R)$ in terms of $w_n(k)$ and $s_n(k)$. In large number of cases we are able to compute $w_n(R)$. Similar results for the nth Waring number of the total ring of fractions of R are obtained. We then provide applications. In particular we compute $w_n(\mathbb{Z}_p)$ and $w_n(\mathbb{Q}_p)$ for $n \in \{3, 4, 5\}$ and any prime p.

Joint work with Piotr Miska.

(24) **Noah Kravitz**, Princeton University
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Title: Lonely Runner spectra, revisited
Abstract: Dirichlet's Theorem says that for any real number t, there is some v in $\{1, 2, ..., n\}$ such that tv is within $1/(n+1)$ of an integer. The Lonely Runner Conjecture of Wills and Cusick asserts that the constant $1/(n+1)$ in this theorem cannot be improved by replacing $\{1, 2, ..., n\}$ with a different set of n nonzero real numbers. The conjecture, although now more than 50 years old, remains wide open for n larger than 6. In this talk I will describe the "Lonely Runner spectra" that arise when one considers the "inverse problem" for the Lonely Runner Conjecture, and I will explain the (a priori surprising) "hierarchical" relations among these spectra.
Joint work with Vikram Giri.

(25) **Andrzej Kukla**, Jagiellonian University in Kraków, Poland
Email: andrzej.kukla@student.uj.edu.pl
Title: Practical sets and A-practical numbers
Abstract: Set $A \subset \mathbb{Z}_+$ is said to be practical if and only if all positive integers smaller than $\sum_{a \in A} a$ are representable as some sum of distinct elements from A. On the basis of this concept we define A-practical numbers, which generalize the notion of practical numbers. In the talk I will discuss basic properties of practical sets and their connection to practical numbers, but also basic results concerning A-practical numbers with emphasis on expansion and removal theorems.

Joint work with Maciej Ulas and Piotr Miska, Jagiellonian University.

(26) **Angel Kumchev**, Towson University
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Title: New versions of old results on gaps between squarefree integers
Abstract: The study of the shortest interval $(x, x+h]$ that contains squarefree integers for all sufficiently large x goes back to the 1940s. A period of active development in the 1980s and early 1990s culminated in a celebrated result of Filaseta and Trifonov that if c is a sufficiently large constant, one can take $h \leq cx^{1/5} \log x$. However, since then progress has stalled, and this is still the best known upper bound on the maximum gap between squarefree integers.

In this talk, I will report on some related work completed by an REU group I supervised jointly with Nathan McNew in the summer of 2022. First, through a thorough examination and careful optimization of the techniques of Filaseta and Trifonov we were able to prove a fully explicit version of their gap result: The interval $(x, x + 11x^{1/5} \log x]$ contains a squarefree integer for all $x \geq 2$. We also proved versions with smaller explicit constants that hold when $x \geq x_0$, where x_0 is given explicitly. The research team proved also such results on gaps between cubefree and general k-free integers by developing ideas of Trifonov from the mid 1990s.

Further, we are able to establish analogs of several bounds on the gaps between squarefree integers for squarefree polynomials over a finite field $\mathbb{F}_q$. In this context, Keating and Rudnick have shown that when $q \to \infty$ one can prove an essentially best possible result. However, the case when q is fixed and the size of the polynomials is determined by their degrees does not appear to have been studied. We show that in that case, the problem resembles the classical problem much more closely. In particular, we prove a variant of the Filaseta-Trifonov theorem for polynomials over finite fields.

(27) **Jeffrey C. Lagarias**, University of Michigan
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Title: The floor quotient partial order
The floor quotient order is given by a relation on the positive integers which says d is less than n if $d = \lfloor n/k \rfloor$ for some integer k. This relation defines a partial order. This talk reports on properties of the partial order restricted to order intervals $[d, n]$, with emphasis on the initial intervals $[1, n]$. These include results on the size of intervals, the internal structure of intervals, and the two-variable Mobius function of the order on intervals. The initial interval has extra structure arising from an involution $d \to \lfloor n/d \rfloor$ acting on its elements. The restriction to the "top half" of these elements has Móbius function related to the usual divisibility Möbius function. Experimentally $\mu(1, n)$ may sometimes take large values. We bound its size and show it has infinitely many sign changes.

Joint work with D. Harry Richman (U. Washington).

(28) **Noah Lebowitz-Lockard**, University of Texas, Tyler, TX
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Title: Long runs of numbers with the same number of divisors
Abstract: In 1952, Erdős and Mirsky asked for an upper bound on the longest sequence of consecutive numbers $\leq x$ with the same number of divisors. No one made any progress until this past January, when Spătaru found the first non-trivial upper bound. We discuss a slight improvement on Spătaru's bound and discuss the analogous problem for related functions, such as the sum-of-proper-divisors function and the Carmichael function. Joint work with Joseph Vandehey.

(29) **Paolo Leonetti**, Universitá degli Studi dell'Insubria, Varese, Italy
Email: leonettipaolo@gmail.com
Title: How many sets of integers have a small perturbation of the type $A + B$?
Abstract: We show, from a topological and from a measure viewpoint, that most sets of nonnegative integers have the property that all their "small" perturbations *cannot* be written as $A+B$, for some $A, B \subseteq \mathbb{N}$ with $|A|, |B| \geq 2$.
References: P.L., *Almost all sets of nonnegative integers and their small perturbations are not sumsets*, Proc. Amer. Math. Soc., to appear.

(30) **Jared Duker Lichtman**, University of Oxford, UK
Email: jared.d.lichtman@gmail.com
Title: Primes in large arithmetic progressions and smooth shifted primes
Abstract: We prove the infinitude of shifted primes $p - 1$ without prime factors above $p^{0.2844}$. This refines $p^{0.2961}$ from Baker and Harman in 1998. Consequently, we obtain an improved lower bound on the distribution of Carmichael numbers. Our main technical result is a new mean value theorem for primes in arithmetic progressions to large moduli. Namely, we estimate primes of size x with quadrilinear forms of moduli up to $x^{0.5312}$. This result extends moduli beyond $x^{0.5233}$ due to Zhang and Polymath, and $x^{0.5238}$ recently obtained by Maynard. These results all build on well-known 1986 work of Bombieri, Friedlander, and Iwaniec.

(31) **Florian Luca**, University of the Witwatersrand, South Africa
Email: florian.luca@wits.ac.za
Title: Carmichael numbers of the form $2^n k + 1$
Abstract: In the first part of the talk we will survey some results concerning Carmichael numbers of the form $2^n k + 1$. Then we will present the proof of the fact that there is no Carmichael number of the above form with k prime.
Joint work with A. Alahmadi.

(32) **Ariane Masuda**, New York City College of Technology (CUNY)
Title: Involutions over finite fields
Email: amasuda@citytech.cuny.edu
Abstract: In the context of memory-limited environments, involutions are desirable because they allow for efficient use of limited memory resources. Specifically, in many applications, both the permutation and its inverse must be stored in memory, which can be a challenge in resource-constrained environments. By using an involution as the interleaver, the same structure and technology used for encoding can be used for decoding as well. Fixed points are points that remain unchanged under the permutation. In cryptographic applications, such as the design of S-boxes, it is desirable to have permutations with a small number of fixed points to increase the security of the system. Therefore, understanding the number of fixed points and how to construct involutions with few fixed points is an important area of research. In the talk I will present some new families of involutions over finite fields, including explicit constructions based on a prescribed number of fixed points.

Part of this work is joint with Juliane Capaverde and Virgínia Rodrigues. Another part is joint with Lillian González-Albino and Ivelisse Rubio.

(33) **Karyn McLellan**, Mount Saint Vincent University
Email: karyn.mclellan@msvu.ca
Title: Counting n^{th} differences of the floor of an irrational sequence
Abstract: Consider the following open problem: Let r be an irrational number with fractional part between $\frac{1}{3}$ and $\frac{1}{2}$. Let C_n be the number of distinct n^{th} differences of the sequence $(\lfloor kr \rfloor)$, where $k \in \mathbb{Z}^+$. Prove or disprove that $C_n = (2, 3, 3, 5, 4, 7, 5, 9, 6, 11, 7, 13, 8, 15, \ldots)$, which is simply a *riffle* of $(2, 3, 4, 5, 6, \ldots)$ and $(3, 5, 7, 9, 11, \ldots)$. In this talk I will discuss the problem and provide some interesting results.

Joint work with Danielle Cox and Shayne Breen, MSVU.

(34) **Nathan McNew**, Towson University
Email: nmcnew@towson.edu
Title: The distribution of intermediate prime factors
Abstract: Let $P^{(\frac{1}{2})}(n)$ denote the middle prime factor of n (taking into account multiplicity). It has previously been shown that $\log\log P^{(\frac{1}{2})}(n)$ has normal order $\frac{1}{2}\log\log x$, and its values follow a Gaussian distribution around this value. We extend this work by obtaining an asymptotic formula for the count of $n \leq x$ for which $P^{(\frac{1}{2})}(n) = p$, for primes p in a wide range up to x. We also generalize the "middle prime factor" to any percentile $\alpha \in (0, 1)$ (and find that the results are subtly different when α is irrational). This result has a variety of applications, for example we can use it to obtain an asymptotic expression for the geometric mean of the middle prime factors, and discover that the golden ratio makes a surprising appearance. We also find that $P^{(\alpha)}(n)$ obeys Benford's leading digit law, and that $P^{(\alpha)}(n)$ is equidistributed in coprime residue classes, in an essentially optimal range of uniformity in the modulus.

Joint work with Paul Pollack and Akash Singha Roy at University of Georgia.

(35) **Piotr Miska**, Jagiellonian University in Kraków, Poland
Email: piotr.miska@uj.edu.pl
Title: On Waring numbers of henselian rings I
Abstract: Let $n > 1$ be a positive integer and R be a commutative ring with unity. We define the nth Waring number $w_n(R)$ as the smallest positive integer g such that every sum of nth powers of elements of R can be written as a sum of at most g nth powers of elements of R.

Let R be a henselian local ring with residue field k of nth level $s_n(k)$. We give some upper and lower bounds for the nth Waring number $w_n(R)$ in terms of $w_n(k)$ and $s_n(k)$. In large number of cases we are able to compute $w_n(R)$. Similar results for the nth Waring number of the total ring of fractions of R are obtained. We then provide applications. In particular we compute the Waring numbers of rings of p-adic integers and fields of p-adic numbers for $n \in \{3, 4, 5\}$ and any prime p.

Joint work with Tomasz Kowalczyk.

(36) **Md Ibrahim Molla**, Ramakrishna Mission Vivekananda Educational and Research Institute (RKMVERI), India
Email: mdibimya@gmail.com
Title: Some zero-sum constants and their associated inverse problems
Abstract: Let G be a finite abelian group (written additively) of exponent m and $A \subset [1, m-1]$ a non-empty set. Then the *Davenport constant of G with weight A,* denoted by $D_A(G)$, is defined to be the smallest positive integer k such that every sequence $S = (x_1, \ldots, x_k)$ over G of length k has a non-empty *A-weighted zero-sum subsequence,* that is, there exist a non-empty subsequence $(x_{j_1}, \ldots, x_{j_t})$ of S and $a_1, \ldots, a_t \in A$ such that $\sum_{i=1}^{t} a_i x_{j_i} = 0$, where 0 is the identity element of G. Similarly, for any such weight set A and for a finite abelian group G of order n, *the constant $E_A(G)$* is defined to be the least $t \in \mathbb{N}$ such that for any sequence $(x_1, \ldots, x_t)$ of t elements with $x_i \in G$, there exists an A-weighted zero-sum subsequence of length n.

The case $A = \{1\}$ corresponds to the classical Davenport constant $D(G)$ and $E(G)$.

For a particular $D_A(G)$ (respectively, $E_A(G)$), the associated *inverse problem* refers to the investigation of *all* sequences over G of length $D_A(G)-1$ (respectively, $E_A(G)$-1) not having any non-empty A-weighted zero-sum subsequence of any length (respectively, of length n).

The study of the inverse problems corresponding to different zero-sum constants with various weight sets is a fascinating topic and has been of growing interest. In this talk, our plan is to discuss some results in this direction.

(37) **Mel Nathanson**, Lehman College (CUNY)
Email: melvyn.nathanson@lehman.cuny.edu
Title: Patterns in the iteration of an arithmetic function
Abstract: Let Ω be a set of positive integers and let $S : \Omega \to \Omega$ be an arithmetic function. Let $V = (v_i)_{i=1}^{n}$ be a finite sequence of positive integers. An integer $m \in \Omega$ has *increasing-decreasing pattern* V with respect to S if, for all odd integers $i \in \{1, \ldots, n\}$,

$$S^{v_1+\cdots+v_{i-1}}(m) < S^{v_1+\cdots+v_{i-1}+1}(m) < \cdots < S^{v_1+\cdots+v_{i-1}+v_i}(m)$$

and, for all even integers $i \in \{2, \ldots, n\}$,

$$S^{v_1+\cdots+v_{i-1}}(m) > S^{v_1+\cdots+v_{i-1}+1}(m) > \cdots > S^{v_1+\cdots+v_{i-1}+v_i}(m).$$

The arithmetic function S is *wildly increasing-decreasing* if, for every finite sequence V of positive integers, there exists an integer $m \in \Omega$ such that m has increasing-decreasing pattern V with respect to S. It is proved that the Collatz function is wildly increasing-decreasing.

(38) **Miquel Ortega**, Universitat Politècnica de Catalunya, Barcelona, Spain
Email: miquel.ortega.sanchez-colomer@upc.edu
Title: Product-free sets in the free group
Abstract: We prove that product-free sets of the free group over a finite alphabet have maximum density $1/2$ with respect to the natural measure that assigns total weight one to each set of irreducible words of a given length. This confirms a conjecture of Leader, Letzter, Narayanan and Walters. In more general terms, we actually prove that strongly k-product-free sets have maximum density $1/k$ in terms of the said measure. The bounds are tight.
Joint work with Juanjo Rué and Oriol Serra.

(39) **Cormac O'Sullivan**, Bronx Community College (CUNY)
Email: cormac.osullivan@bcc.cuny.edu
Title: A hidden link between two of Ramanujan's approximations
Abstract: In consecutive notebook entries, Ramanujan gave asymptotic approximations to the exponential function and the exponential integral. The asymptotic expansion coefficients seem to agree up to an alternating sign, as we conjectured in an earlier paper. We establish this hidden link with a combinatorial proof that involves Stirling numbers, second-order Eulerian numbers, modifications of both of these, and Stirling's approximation to the gamma function. An analytic second proof has also been provided by a referee. It is not yet clear if this result is an oddity or part of a broader picture.

(40) **Péter Pál Pach**, Budapest University of Technology and Economics, Hungary
Email: p.p.pach@gmail.com
Title: The Alon-Jaeger-Tarsi conjecture via group ring identities
Abstract: The Alon-Jaeger-Tarsi conjecture states that for any finite field $\mathbb{F}$ of size at least 4 and any nonsingular matrix A over $\mathbb{F}$ there exists a vector x such that neither x nor Ax has a 0 component. In this talk we discuss the proof of this result for sufficiently large primes and further applications of our method about coset covers and additive bases.

Joint work with János Nagy and István Tomon.

(41) **Firdavs Rakhmonov**, University of Rochester
Email: frakhmon@ur.rochester.edu
Title: Distribution of similar configurations in subsets of $\mathbb{F}_q^d$
Abstract: Let $\mathbb{F}_q$ be a finite field of order q and E be a set in $\mathbb{F}_q^d$. The distance set of E is defined by $\Delta(E) := \{\|x-y\| : x, y \in E\}$, where $\|\alpha\| = \alpha_1^2 + \cdots + \alpha_d^2$. Iosevich, Koh and Parshall (2018) proved that if $d \geq 2$ is even and $|E| \geq 9q^{d/2}$, then

$$\mathbb{F}_q = \frac{\Delta(E)}{\Delta(E)} = \left\{ \frac{a}{b} : a \in \Delta(E),\ b \in \Delta(E) \setminus \{0\} \right\}.$$

In other words, for each $r \in \mathbb{F}_q^*$ there exist $(x, y) \in E^2$ and $(x', y') \in E^2$ such that $\|x - y\| \neq 0$ and $\|x' - y'\| = r\|x - y\|$.

Geometrically, this means that if the size of E is large, then for any given $r \in \mathbb{F}_q^*$ we can find a pair of edges in the complete graph $K_{|E|}$ with vertex set E such that one of them is dilated by $r \in \mathbb{F}_q^*$ with respect to the other. A natural question arises whether it is possible to generalize this result to arbitrary subgraphs of $K_{|E|}$ with vertex set E and this is the main goal of the talk.

In this talk, Ill explain the proof of the problem for k-paths ($k \geq 2$), simplexes and 4-cycles. The proof is based on a mixture of tools from different areas such as enumerative combinatorics, group actions and Turn type theorems.

(42) **Alex Rice**, Millsaps College
Email: riceaj@millsaps.edu
Title: Generalized arithmetic progressions and diophantine approximation by polynomials
Abstract: We discuss two related notions of "approximate subgroups" inside finite sets of integers: Bohr sets, which capture simultaneous Diophantine approximation, and symmetric generalized arithmetic progressions (GAPs). For example, fix natural numbers N and d and consider the following pair of questions:

(a) For fixed $\alpha_1, \ldots, \alpha_d \in \mathbf{R}$, how close to an integer can we simultaneously make $n^2\alpha_1, \ldots, n^2\alpha_d$ for some $1 \leq n \leq N$?
(b) How large can a set of the form $\{x_1\ell_1 + \cdots + x_d\ell_d : |\ell_i| \leq L_i\} \subseteq [-N, N]$ be before it is guaranteed to contain a perfect square?

Our discussions range from classical facts like the Kronecker approximation theorem and Linnik's theorem, to a recent breakthrough result of Maynard and its potential future applications. In between we survey results including previous joint work with Neil Lyall and Ernie Croot.

(43) **Asher Roberts**, CUNY Graduate Center
Email: aroberts@gradcenter.cuny.edu
Title: The rate of convergence for Selberg's multivariate CLT
Abstract: We discuss the approach of Radziwiłł and Soundararajan to proving Selberg's central limit theorem for the Riemann zeta function on the critical line. We will see that this approach can be extended to a multivariate context, providing another proof of a theorem of Bourgade. By tracking the error resulting from each step, with the help of a carefully chosen metric, we obtain a rate of convergence for both the original theorem and the multivariate extension.

(44) **David A. Ross**, University of Hawai'i
Email: ross@math.hawaii.edu
Title: Some proofs about sequences in the spirit of Paul du Bois-Reymond
Abstract: Across several papers in the nineteenth century, du Bois-Reymond developed a 'calculus' of scales of infinity, that is, relative sizes in the limit of functions increasing to infinity. Other mathematicians—notably Hadamard, Borel, and Hardy—later developed this machinery further. I'll apply these ideas to three examples: a Fekete-style theorem; a result about almost additive sequences due to Pólya and Szegő; and an asymptotic fixed point theorem.

(45) **Misha Rudnev**, University of Bristol, UK
Email: m.rudnev@bristol.ac.uk
Title: On sums and products of integers with few prime factors
Abstract: I will sketch the proof of a new sum-product result showing that for a set A of N integers, each of which has o(log N) prime factors, either the product set A*A is big or there is a large subset with small additive energy. The proof relies on martingale maxmal inequalities and Schmidt's subspace theorem. The bound is optimal, up to sub-polynomial factors of N, matching the example by Balog and Wooley.
This is joint work with Hanson, Shkredov and Zhelezov.

(46) **Anurag Sahay**, University of Rochester
Email: anuragsahay@rochester.edu
Title: The VC-dimension of multiplicative structures in finite fields
Abstract: Let $\Gamma \subseteq \mathbb{F}_q^\times$ be a multiplicative subgroup of a finite field. We consider the VC dimension of the set system $\mathcal{H} = \{\Gamma + x : x \in \mathbb{F}_q\}$ consisting of its additive translates. An example of particular interest is the set of quadratic residues, $\Gamma = \mathcal{S}_q = \{x^2 : x \in \mathbb{F}_q^\times\}$.

We present some conjectures regarding this VC-dimension; the basis for these conjectures is the expectation that additive and multiplicative structure in finite fields must be loosely correlated. We also provide some partial progress towards our conjectures. The main technical tool is the Weil bound for multiplicative character sums.

Joint work with Brian McDonald and Emmett Wyman.

(47) **Aliaksei Semchankau**, Carnegie Mellon University
Email: asemchan@andrew.cmu.edu
Title: On the sequence $n!$ mod p
Abstract: Let p be a prime, and let $\mathcal{A}$ be a set of residues of the sequence $1!, 2!, 3!, \ldots, (p-1)!$ modulo p. We prove

$$|\mathcal{A}| \geqslant (\sqrt{2} + o(1))\sqrt{p}.$$

Now consider an interval $\mathcal{I} \subseteq \{0, 1, \ldots, p-1\}$ of length $N > p^{7/8+\varepsilon}$ and denote by $\mathcal{A}_{\mathcal{I}}$ the set of residues modulo p it produces. Then we prove

$$|\mathcal{A}_{\mathcal{I}}| > (1 + o(1))\sqrt{p}.$$

Tools used are results from Algebraic Geometry as black boxes and simple combinatorial observations.

This is joint work with Alexandr Grebennikov, Arsenii Sagdeev, Aliaksei Vasilevskii, and available on arXiv.

(48) **Steven Senger**, Missouri State University
Email: stevensenger@gmail.com
Title: Recent developments in point configuration problems in vector spaces over various fields
Abstract: We discuss recent results involving the ubiquity of distinct isomorphic copies of point configurations in vector spaces over fields, both finite and infinite. This work is inspired by the results in the fractal setting on trees due to Yumeng Ou and Krystal Taylor.

(49) **Sayak Sengupta**, SUNY-Binghamton
Email: ssengup1@binghamton.edu
Title: Locally nilpotent polynomials over $\mathbf{Z}$
Abstract: Let K be a number field and $\mathcal{O}_K$ be the ring of integers of K. For a polynomial $u(x)$ in $\mathcal{O}_K[x]$ and $r \in \mathcal{O}_K$, we can construct a dynamical sequence $u(r), u^{(2)}(r), \ldots$. Let $P(u^{(n)}(r)) := \{\mathfrak{p} \in \mathrm{MSpec}(\mathcal{O}_K) \mid u^{(n)}(r) \in \mathfrak{p}, \text{for some } n \in \mathbb{N}\}$. For which polynomials $u(x)$ and $r \in \mathcal{O}_K$ do we expect to have $P(u^{(n)}(r)) = \mathrm{MSpec}(\mathcal{O}_K)$? If we hit 0 somewhere in the above sequence, then we obviously have the equality. If we do not hit zero for any iteration then the question becomes very interesting. In this talk, we will define such polynomials for a general number field K and then we will look at some results in the particular case of $K = \mathbb{Q}$.

(50) **Adam Sheffer**, Baruch College (CUNY)
Email: adamsh@gmail.com
Title: Distinct distances in the complex plane
Abstract. We prove that if $\mathcal{P}$ is a set of n points in $\mathbb{C}^2$, then either the points in $\mathcal{P}$ determine $\Omega(n^{1-\varepsilon})$ complex distances, or $\mathcal{P}$ is contained in a line with slope $\pm i$. If the latter occurs, then each pair of points in $\mathcal{P}$ have complex distance 0.
Joint work with Joshua Zahl.

(51) **Liyang Shen**, New York University
Email: Liyang.Shen@nyu.edu
Title: Linear recurrences of order at most two in nontrivial small divisors and large divisors
Abstract: For each positive integer N, define

$$S'_N = \{1 < d < \sqrt{N} : d|N\} \text{ and } L'_N = \{\sqrt{N} < d < N : d|N\}.$$

Recently, Chentouf characterized all positive integers N such that the set of small divisors $\{d \leq \sqrt{N} : d|N\}$ satisfies a linear recurrence of order at most two. We nontrivially extend the result by excluding the trivial divisor 1 from consideration, which dramatically increases the analysis complexity. Our first result characterizes all positive integers N such that S'_N satisfies a linear recurrence of order at most two. Moreover, our second result characterizes all positive N such that L'_N satisfies a linear recurrence of order at most two, thus extending considerably a recent result that characterizes N with L'_N being in an arithmetic progression.

Joint work with Hùng Việt Chu, Kevin Huu Le, and Steven J. Miller, Yuan Qiu.

(52) **I.D. Shkredov**, Steklov Mathematical Institute, Moscow, Russia
Email: ilya.shkredov@gmail.com
Title: On some multiplicative properties of large difference sets
Abstract: We study multiplicative properties of difference sets $A - A$ for large sets $A \subseteq \mathbf{Z}/q\mathbf{Z}$ in the case of composite q. A quantitative version of a result of A. Fish about the structure of the product sets $(A - A)(A - A)$ is obtained. We show that the multiplicative covering number of any difference set is always small. Also, we find several applications of our results to various sums with multiplicative characters.

(53) **Satyanand Singh**, New York College of Technology (CUNY)
Email: ssingh@citytech.cuny.edu
Title: On the Pythagorean reciprocal identity $a^{-2} + b^{-2} = d^{-2}$ and the generation of all solutions for fixed $d > 0$
Abstract: The Pythagorean reciprocal identity originates by way of a special perpendicular construct in a right triangle. Roger Vogel in 1999 showed how to generate solutions to $a^{-2} + b^{-2} = d^{-2}$, with $\gcd(a, b, d) = 1$, via the primitive triples of the right triangle. We remove the requirement that $\gcd(a, b, d) = 1$ and show that this diophantine equation has solutions if and only if $d \equiv 0 \mod 12$. For fixed d, we generate all solutions to the identity by two different methods. Our techniques employ appropriate bounds on

the variables and one of them is connected to the OEIS sequence A020884. As an example, for $d = 120$, $(a, b) \in \{(130, 312), (136, 225), (150, 200)\}$ which as we will see results in three distinct triangles. You can switch the order of a and b to get three additional algebraic solutions. This example refers to a problem I created for the MAA Metro NY problem of the month for March 2023.
This is joint work with Alexander Rozenblyum.

(54) **Bartosz Sobelewski**, Jagiellonian University, Kraków, Poland
Email: bartosz.sobelewski@uj.edu.pl
Title: Values of the colored binary partition function represented as a sum of three squares
Abstract: For a fixed positive integer m, let $b_m(n)$ denote the number of partitions of $n \in \mathbb{N}$ with parts being powers of 2, where each part can take one of m colors. In particular, b_1 is the classical binary partition function, already studied by Euler, while the case $m \geq 2$ was introduced rather recently. We are interested in the existence of a representation

$$b_m(n) = x^2 + y^2 + z^2,$$

where $x, y, z \in \mathbb{Z}$. In the case $m = 2^k - 1$ we give a "nice" characterization of the set S_m of n such that this equation has a solution, which involves terms of the Thue–Morse sequence. This allows us to calculate the natural density of S_m and provide an accurate approximation of its counting function. We also discuss the harder case $m \neq 2^k - 1$ and present some numerical results concerning related equations.

Joint work with Maciej Ulas.

(55) **Gábor Somlai**, ELTE/CUNY Graduate Center
Enmail: zsomlei@gmail.com
Title: Cutting the positive real numbers into disjoint sets closed under addition and multiplication
Abstract: Róbert Freud asked whether the positive real numbers can be cut into two disjoint sets both closed under addition and multiplication. Even if it might sound surprising, there is such a decomposition. This is mostly due to the fact that $\mathbb{Q}(\alpha)$ can be cut into two pieces both closed under addition and multiplication, if α is transcendental. However, any finite extension of $\mathbb{Q}$ is indecomposable.

The tools used are just a combination of elementary ideas from geometry, analysis and c

Joint work with Gergely Kiss and Tamás Terpai.

(56) **Christoph Spiegel**, Zuse Institute Berlin, Germany
Email: spiegel@zib.de
Title: Towards flag algebras in additive combinatorics
Abstract: We study an analogue of the Ramsey multiplicity problem for additive structures, in particular establishing the minimum number of monochromatic 3-APs in 3-colorings of $\mathbb{F}_3^n$ as well as obtaining the first non-trivial lower bound for the minimum number of monochromatic 4-APs in 2-colorings of $\mathbb{F}_5^n$. The former parallels results by Cumings et al (2013) in extremal

graph theory and the latter improves upon results of Saad and Wolf (2017) The lower bounds are notably obtained by extending the flag algebra calculus of Razborov (2007) to additive structures in vector spaces over finite fields.

This is joint work with Juanjo Ru and available on arXiv: https://arxiv.org/abs/2304.00400

(57) **Christian Táfula**, Université de Montréal, Canada
Email: christian.tafula.santos@umontreal.ca
Title: On the structure of t-representable sumsets
Abstract: Let $A \subseteq \mathbb{Z}_{\geq 0}$ be a finite set with minimal element 0, maximum element m, and ℓ elements in between. Write $(hA)^{(t)}$ for the set of integers that can be written in at least t ways as a sum of h elements of A. In 1970, Nathanson showed that $hA = (hA)^{(1)}$ enjoys a notion of "structure" for large h, allowing us to determine hA in a relatively simple fashion. This was revisited by Granville, Shakan and Walker in 2020, who showed that hA is structured for every $h \geq m - \ell$, and that finite sets in $\mathbb{Z}^d$ also have a similar notion of structure.

Also in 2020 (50 years later!), Nathanson showed that $(hA)^{(t)}$ enjoys a notion of structure as well for large h. In this talk, we will study the problem of giving upper bounds to how large h should be, showing that $(hA)^{(t)}$ is structured if $h \gtrsim \frac{1}{e} m\ell t^{1/\ell}$. This estimate is asymptotically sharp, as we can construct a family of sets $A = A(m, \ell, t) \subseteq \mathbb{Z}_{\geq 0}$ for which $(hA)^{(t)}$ is not structured for $h \leq (1 - o(1)) \frac{1}{e} m\ell t^{1/\ell}$.

If time allows, we will also discuss how $(hA)^{(t)}$ for finite sets $A \subseteq \mathbb{Z}^d$ enjoys a similar notion of structure for large h.

(58) **Ruiwen (Raven) Tang**, Stuyvesant High School
Email: ruiwentang@gmail.com
Title: The constant of point-line incidence constructions
Abstract: We study a lower bound for the constant of the Szemerédi–Trotter theorem. In particular, we show that a recent infinite family of point-line configurations satisfies $I(\mathcal{P}, \mathcal{L}) \geq (c + o(1))|\mathcal{P}|^{2/3}|\mathcal{L}|^{2/3}$, with $c \approx 1.27$. Our technique is based on studying a variety of properties of Euler's totient function. We also improve the current best constant for Elekes's construction from 1 to about 1.27.
Joint work with Martin Balko and Adam Sheffer.

(59) **Marc Technau**, Graz University of Technology, Austria
Email: mtechnau@math.tugraz.at
Title: On the Farey fraction spin chain
Abstract: In 1999, Kleban and Özlük introduced a 'Farey fraction spin chain' and made a conjecture regarding its asymptotic number of states with given energy, the latter being given (up to some normalisation) by the number $\Phi(N)$ of 2×2 matrices arising as products of $\left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ whose trace equals N. Although their conjecture was disproved by Peter (2001),

quite precise results are known on average by works of Kallies–Özlük–Peter–Snyder (2001), Boca (2007) and Ustinov (2013). We shall attempt to convey some of the ideas underlying the above works.

Precise asymptotics for $\Phi(N)$ had hitherto only been known conditionally on the availability of zero-free strips for certain Dirichlet L-functions. However, in this talk we shall see that the question regarding asymptotics for $\Phi(N)$ can be reduced to a problem studied (and solved!) much earlier by Hooley (1958) in a special case and, quite recently, in full generality by Bykovskiĭ and Ustinov (2019). The reduction itself is short and completely elementary.

(60) **Ognian Trifonov**, University of South Carolina
Email: trifonov@math.sc.edu
Title: Representing positive integers as a sum of a squarefree number and a small prime
Abstract: We prove that every positive integer n which is not equal to 1, 2, 3, 6, 11, 30, 155, or 247 can be represented as a sum of a squarefree number and a prime not exceeding $\sqrt{n}$.

Joint work with Jack Dalton.

(61) **Tim Trudgian**, UNSW Canberra at the Australian Defence Force Academy
Email: t.trudgian@adfa.edu.au
Title: A hair's breadth of a half
Abstract: The now-classical method of Montgomery and Odlyzko shows that there are infinitely many pairs of zeroes of the zeta-function closer than the average spacing. It has been known for almost 40 years that this method cannot show an infinitude of gaps less than half of the average spacing. But could it show infinitely many gaps of exactly half the average spacing? This is connected to the question of existence of Siegel zeroes. I shall outline the history of this problem, and show that even getting very close to one half is not possible by this method.

Joint work with Dan Goldston (San Jose State University) and Caroline Turnage-Butterbaugh (Carleton College).

(62) **Daniel Tsai**, National Taiwan University,Taiwan (R.O.C.)
Email: tsaidaniel@ntu.edu.tw
Abstract: We define an additive function $v\colon \mathbb{N} \to \mathbb{Z}$ by setting $v(p) = p$ for primes p and $v(p^\alpha) = p + \alpha$ for prime powers p^α with $\alpha \geq 2$. Similar functions such as those from A008474, A001414, and A000026 in the OEIS have been studied. Denote by $r_b(n)$ the integer formed by reversing the base b digits of an integer $n \geq 1$; we write $r(n)$ for $r_{10}(n)$. For example, if $n = 18$, then $r(18) = 81$ but $r_2(18) = r_2(10010_2) = 1001_2 = 9$. Spiegelhofer has obtained a family of pairs (f, b), where $f\colon \mathbb{N} \to \mathbb{C}$ and $b \geq 2$, such that $f(n) = f(r_b(n))$ for all $n \geq 1$.

Considering the equality $v(n) = v(r(n))$, we say an integer $n \geq 1$ is a *v-palindrome* if $10 \nmid n$, $n \neq r(n)$, and $v(n) = v(r(n))$; this generalizes *decimal palindrome*, integers n where $n = r(n)$. Tsai proved the following

sequences are v-palindromes:

$$18, 198, 1998, 19998, 199998, \ldots,$$
$$18, 1818, 181818, 18181818, 1818181818, \ldots.$$

The sequence of v-palindromes is A338039 in the OEIS.

It was conjectured that no prime v-palindromes exist. We prove that prime v-palindromes are precisely the primes of the form $5 \cdot 10^m - 1$ such that $5 \cdot 10^m - 3$ is also prime, and thus standard heuristics suggests that there are only finitely many prime v-palindromes.

Joint work with Muhammet Boran, Steven J. Miller, Jesse Purice, and Garam Choi.

(63) **Maciej Ulas**, Jagiellonian University, Kraków, Poland
Email: maciel.ulas@gmail.com
Title: Geometric progressions in the sets of values of rational functions
Abstract: Let $a, Q \in \mathbf{Q}$ be given and consider the set $\mathcal{G}(a, Q) = \{aQ^i : i \in \mathbf{N}\}$ of terms of geometric progression with 0th term equal to a and the quotient Q. Let $f \in \mathbf{Q}(x, y)$ and $\mathcal{V}_f$ be the set of finite values of f. We consider the problem of existence of $a, Q \in \mathbf{Q}$ such that $\mathcal{G}(a, Q) \subset \mathcal{V}_f$. In the first part of the talk we describe several classes of rational function for which our problem has a positive solution. In particular, if $f(x, y) = \frac{f_1(x,y)}{f_2(x,y)}$, where $f_1, f_2 \in \mathbf{Z}[x, y]$ are homogenous forms of degrees d_1, d_2 and $|d_1 - d_2| = 1$, we prove that $\mathcal{G}(a, Q) \subset \mathcal{V}_f$ if and only if there are $u, v \in \mathbf{Q}$ such that $a = f(u, v)$. In the second, experimental, part of the talk we study the stated problem for the rational function $f(x, y) = (y^2 - x^3)/x$. We relate the problem to the existence of rational points on certain elliptic curves and present interesting numerical observations which allow us to state several questions and conjectures.

(64) **Aled Walker**, King's College London, UK
Email: aled.walker@kcl.ac.uk
Title: On a Bohr set analogue of Chowla's conjecture
Abstract: Let λ denote the Liouville function. In 1965, Chowla made a conjecture on the natural average of the sequence $\lambda(n)\lambda(n + 1)$ (and more generally the average of $\lambda(f(n))$ for polynomial f). From this the subject has recently expanded, and there are now a suite of conjectures on the natural averages (and logarithmic averages) of sequences $\lambda(a_1(n)) \cdots \lambda(a_k(n))$, where $a_1(n), \ldots, a_k(n)$ are certain arithmetic sequences. In this talk we will discuss recent joint work with Joni Teräväinen, in which we established that the logarithmic average of $\lambda(\lfloor \alpha_1 n \rfloor)\lambda(\lfloor \alpha_2 n \rfloor)$ is 0 whenever α_1, α_2 are positive reals with α_1/α_2 irrational. We will also prove non-trivial cancellation on the logarithmic average of $\lambda(\lfloor \alpha_1 n \rfloor) \cdots \lambda(\lfloor \alpha_k n \rfloor)$ when $k \geqslant 3$. These results answer the two-point case of a conjecture of Frantzikinakis, and provide some progress on the higher order cases.

(65) **Trevor Wooley**, Purdue University
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Title: Waring's problem and Freĭman's theorem
Abstract: Freĭman proved that when (k_i) is an increasing sequence of positive integers, then for each j, there exists $s = s(j)$ having the property that all large integers n are represented as a sum of positive integral k_i-th powers (with $i \in \{j, j+1, \ldots, s\}$) if and only if the series $1/k_1 + 1/k_2 + \ldots$ diverges. We describe recent work joint with Jörg Brüdern making Freĭman's theorem effective. Some concrete examples will be described.

(66) **Kiseok Yeon**, Purdue University
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Title: The Hasse principle for homogeneous polynomials with random coefficients over thin sets
Abstract: In this talk, we introduce a framework via the circle method in order to confirm the Hasse principle for random projective hypersurfaces over $\mathbb{Q}$. We first give a motivation for developing this framework by providing an overall history of the problems of confirming the Hasse principle for projective hypersurfaces over $\mathbb{Q}$. Next, we provide a sketch of the proof of our main result and show a part of the estimates used in the proof. Furthermore, we introduce an auxiliary mean value theorem which plays a crucial role in our argument and may be of independent interest.