CANT 2024

.

Twenty-second Annual Workshop on Combinatorial and Additive Number Theory

CUNY Graduate Center
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Abstracts

(1) **Sukumar Das Adhikari**, Ramakrishna Mission Vivekananda Educational and Research Institute (RKMVERI), India
Email: adhikarisukumar@gmail.com
Title: Some elementary algebraic and combinatorial methods in the study of zero-sum theorems
Abstract: Originating from a beautiful theorem of Erdos-Ginzberg-Ziv about sixty years ago and some other questions asked around the same time, the area of zero-sum theorems has many interesting results and several unanswered questions.

Several authors have introduced interesting elementary algebraic techniques to deal with these problems. We describe some experiments with these elementary algebraic methods and some combinatorial ones, in a weighted generalization in the area of Zero-sum Combinatorics.

(2) **Adrian Beker**, University of Zagreb, Croatia
Email: adrian.beker@math.hr
Title: On a problem of Erdős and Graham about consecutive sums in strictly increasing sequences
Abstract: Given a finite sequence of integers $a = (a_i)_{1 \le i \le k}$, let $S(a)$ denote the set of its consecutive sums, that is, sums of the form $\sum_{i=u}^{v} a_i$ with $1 \le u \le v \le k$. Erdős and Graham asked whether there exists a constant $c > 0$ such that, for all positive integers n, there is such a sequence in $\{1, \ldots, n\}$ which is strictly increasing and satisfies $|S(a)| \ge cn^2$.

The obvious candidate consisting of all integers from 1 up to n falls short of having this property due to reasons related to the multiplication table problem. On the other hand, if we drop the monotonicity assumption, such sequences were shown to exist by Hegyvári via a construction based on Sidon sets. In this talk, I will present two constructions, one probabilistic and the other deterministic, that give an affirmative answer to the starting question. I will also discuss some non-trivial upper bounds on the size of $S(a)$ in this setting.

(3) **Christine K. Chang**, CUNY Graduate Center
Email: cchang1@gradcenter.cuny.edu
Title: Hybrid statistics of the maxima of a random model of the zeta function over short intervals
Abstract: We will present a matching upper and lower bound for the right

tail probability of the maximum of a random model of the Riemann zeta function over short intervals. In particular, we show that the right tail interpolates between that of log-correlated and IID random variables as the interval varies in length. We will also discuss a new normalization for the moments over short intervals. This result follows the recent work of Arguin-Dubach-Hartung and is inspired by a conjecture by Fyodorov-Hiary-Keating on the local maximum over short intervals.

(4) **Jonathan Chapman**, University of Bristol, UK
Email: jonathan.chapman@bristol.ac.uk
Title: Monochromatic sums and products
Abstract: If we colour $\{2, \dots, N\}$ with r different colours, how many monochromatic solutions to $xy = z$ appear? A classical theorem of Schur shows that we always obtain $(c_r + o(1))N^2$ monochromatic solutions (as $N \to \infty$) to $x + y = z$, for some $c_r > 0$, which is within a constant factor of the total number of solutions. However, Prendiville showed that one cannot achieve such a strong result for $xy = z$, even if one only uses 2 colours.

In this talk, I will present recent work on determining the asymptotic minimum number of monochromatic solutions to $xy = z$. We prove that every 2-colouring of $\{2, \dots, N\}$ produces at least $(2^{-3/2} + o(1))\sqrt{N} \log N$ monochromatic solutions to $xy = z$, and the leading constant is sharp. I will also introduce a Schur-type problem for colourings of real numbers. If the discrete and continuous Schur problems are 'quantitatively equivalent', then, for an arbitrary number of colours, our upper and lower bounds for the number of monochromatic solutions to $xy = z$ match up to a logarithmic factor.

Joint work with Lucas Aragão, Miquel Ortega, and Victor Souza.

(5) **Eric Dolores Cuenca**, Pusan National University, Korea
Email: eric.rubiel@pusan.ac.kr
Title: Zeta values as an algebra over an operad
Abstract: Denote the operad of finite posets by FP. In number theory, the field of rational zeta series studies series of the form $\sum_{i=1}^{\infty} a_i(\zeta(i+1) - 1), a_i \in \mathbb{Q} \, \forall i \in \mathbb{N}$, where $\zeta(k)$ is the Riemann zeta function $\zeta(k) = \sum_{n=1}^{\infty} \frac{1}{n^k}$. By studying zeta values as algebras over the operad of posets, we show the following identity, for $a > 1, a \in \mathbb{N}$:

$$\sum_{n=i}^{\infty} (-1)^{n+1} \binom{n}{i} \zeta(n+1, a) = (-1)^{i+1} \zeta(i+1, a+1),$$

here, $\zeta(k, a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^k}$ is the Hurwitz zeta function.

On January 2023 we put the left side of the identity on several private software, but none of them produced any output. We presented our work in the Wolfram Technology Conference 2023, where their team kindly verified that the left side of the identity is equal to the right side of the identity. Joint work with Jose Mendoza-Cortes, Michigan State University

(6) **James Cumberbatch**, Purdue University
Email: jcumberb@purdue.edu
Title: Digitally restricted sets and the Goldbach conjecture
Abstract: We show that given any base b and any set of digits $\mathcal{D}$ with at least two digits, let $\mathcal{A}$ be the set of integers whose base-b digits consist only of values in $\mathcal{D}$. We prove that almost all even integers in $\mathcal{A}$ are sum of two primes.

(7) **Robert Donley**, Queensborough Community College (CUNY)
Email: rdonley@qcc.cuny.edu
Title: A combinatorial introduction to adinkras
Abstract: In 2005, Faux and Gates defined the adinkra, a graphical device for describing particle exchanges in supersymmetry. Independent of the physical applications, the adinkra resides at a nexus of various mathematical concepts and problems. In this talk, we give an introduction to adinkras from the point of view of matching problems in combinatorics.

Joint work with S. James Gates, Jr., Tristan Hübsch, and Rishi Nath.

(8) **Jin-Hui Fang**, Nanjing Normal University, China
Email: fangjinhui1114@163.com
Title: On Cilleruelo-Nathanson's method in Sidon sets
Abstract: For nonnegative integers h, g with $h \geq 2$, a set $\mathcal{A}$ of nonnegative integers is defined as a $B_h[g]$ sequence if, for every nonnegative integer n, the number of representations of n with the form $n = a_1 + a_2 + \cdots + a_h$ is no larger than g, where $a_1 \leq \cdots \leq a_h$ and $a_i \in \mathcal{A}$ for $i = 1, 2, \cdots, h$. Let $\mathbb{Z}$ be the set of integers and $\mathbb{N}$ be the set of positive integers. In 2013, by introducing the method of *Inserting Zeros Transformation*, Cilleruelo and Nathanson obtained the following nice result: let $f : \mathbb{Z} \to \mathbb{N} \bigcup \{0, \infty\}$ be any function such that $\liminf_{|n|\to\infty} f(n) \geq g$ and let $\mathcal{B}$ be any $B_h[g]$ sequence. Then, for any decreasing function $\epsilon(x) \to 0$ as $x \to \infty$, there exists a sequence $\mathcal{A}$ of integers such that $r_{\mathcal{A},h}(n) = f(n)$ for all $n \in \mathbb{Z}$ and $\mathcal{A}(x) \gg B(x\epsilon(x))$. In 2022, Nathanson further considered Sidon sets for linear forms. Recently, we apply the Inserting Zeros Transformation into Sidon sets for linear forms and generalize the above result related to the inverse problem of representation functions.

(9) **Leonid Fel**, Technion – Israel Institute of Technology, Israel
Email: lfel@cv.technion.ac.il
Title: Ratio between a sum of generators and rational powers of their product
Abstract: We study a ratio $R_m(k) = I_1/\sqrt[k]{I_m}$ between a sum $I_1 = \sum_{j=1}^m d_j$ of generators and rational powers $\sqrt[k]{I_m}$ of their product $I_m = \prod_{j=1}^m d_j$ in numerical semigroups $\langle d_1, \ldots, d_m \rangle$. We find its upper $\mathsf{R}_m^+(k)$ and lower $\mathsf{R}_m^-(k)$ bounds in the range $1 \leq k \leq m$. We prove that $R_m(k)$ has a universal upper bound if and only if $m \geq 2k - 1$.

(10) **Daniel Flores**, Purdue University
Email: flore205@purdue.edu
Title: A circle method approach to K-multimagic squares
Abstract: In this paper we investigate *K-multimagic squares* of order N, which are $N \times N$ magic squares with remain magic after raising each element to the kth power for all $2 \leq k \leq K$. Given $K \geq 2$, we consider the problem of establishing the smallest integer $N(K)$ for which there exists *non-trivial* K-multimagic squares of order $N(K)$. Previous results on multimagic squares show that $N(K) \leq (4K-2)^K$ for large K. Here we utilize the Hardy-Littlewood circle method and establish the bound

$$N(K) \leq 2K(K+1)+1.$$

Via an argument of Granville's we additionally deduce the existence of infinitely many *non-trivial* prime valued K-multimagic squares of order $2K(K+1)+1$.

(11) **Krystian Gajdzica**, Jagiellonian University, Kraków, Poland
Email: krystian.gajdzica@im.uj.edu.pl
Title: A combinatorial approach to the Bessenrodt-Ono type inequalities
Abstract: In 2016, Bessenrodt and Ono showed that the partition function satisfies the inequality of the form

$$p(a)p(b) > p(a+b)$$

for all $a, b \geqslant 2$ with $a+b > 9$. Their proof is based on the asymptotic estimates of $p(n)$ due to Lehmer. Since then, a lot of similar phenomena have been discovered for various variations of the partition function.

We discuss the analogue of the Bessenrodt-Ono inequality for the so-called A-partition function $p_A(n)$, which enumerates those partitions of n whose parts belong to a fixed set $A \subset \mathbb{N}$. Since there is no known asymptotic formula for $p_A(n)$ in general, we can not deal with the problem using any estimates of $p_A(n)$. Therefore, we present a combinatorial approach to the issue by constructing an appropriate injection between some sets of partitions.

(12) **N. Hegyvári**, Eötvös Loránd University and Rényi Institute, Hungary
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Title: On the structures of sets in $\mathbb{N}^k$ having thin subset sums
Abstract: For any $X \subseteq \mathbb{N}^k$ let

$$FS(X) := \{\sum_{i=1}^{\infty} \varepsilon_i x_i : \ x_i \in X, \ \varepsilon_i \in \{0,1\}, \ \sum_{i=1}^{\infty} \varepsilon_i < \infty\}$$

Erdős called a sequence $A \subseteq \mathbb{N}$ *complete* if every sufficiently large number belongs to $FS(A)$. In a higher dimension too, the necessary condition that the subset sums of a subset $X \subseteq \mathbb{N}^k$ represent all far points of $\mathbb{N}^k$ should be the condition $X(N) > k \log_2 N - t_X$ for some t_X, i.e. X is complete respect to the region $R = \{x = (x_1, x_2, \dots, x_k) : x_i \geq r_i\}$, $r_i \in \mathbb{N}$, $i = 1, 2, \dots, k$.

Let $A = \{a_1 < a_2 < \dots < a_n < \dots\}$ be an infinite sequence of integers. We say that A is *weakly thin* if $\limsup_{n\to\infty} \frac{\log a_n}{\log n} = \infty$, or equivalently $A(n) := \sum_{a_i \leq n} 1 = n^{g(n)}$, where $A(n)$ is the counting function of A and

$\liminf_{n\to\infty} g(n) = 0$. A set $B \subseteq \mathbb{N}$ is said to be *thick* if it is not weakly thin. Let $X \subseteq \mathbb{N}^k$. X is said to be *thin complete set* respect to R if $X(N) > k\log_2 R(N) - t_X$ for some t_X and $FS(X) \supseteq R$. We prove that if $R = z + \mathbb{N}^k$ and A is complete with respect to R, then all projections of A onto for all axis f_i are thick. We also determine regions for which there exists thin complete sets. Some related results are also discussed.

This is joint work with Máté Pálfy and Erfei Yue.

(13) **Russell Jay Hendel**, Towson University
Email: RHendel@Towson.edu
Title: Local distance-resistance functions equivalent to global symmetries in electric circuit families
Abstract: In a recent paper Hendel explored the computational attributes of an algorithm introduced by Barrett, Evans, and Francis, which, among other things, studied distance resistance in a family of circuits whose underlying graphs consisted of n rows of upright equilateral triangles (n-grids). Two important conjectures supported by numerical evidence were presented: one related to the asymptotic behavior of iterated use of the algorithm on an initial n grid as n goes to infinity. The second conjecture showed that as n grows large certain limiting ratios emerge among specified edges in the circuits resulting from a large number of repeated applications of the algorithm to an initial n-grid. The purpose of this paper is to provide insight into these asymptotic or limiting edge ratios. After introducing the algorithm and reviewing the original conjectures, the main part of this paper studies a family of n-grids whose edge labels are determined using these limiting edge-ratios functions. The main result proven is that these n-grids as well as the graphs derived from repeated application of the algorithm possess vertical and rotational symmetries and also continue to satisfy the relationships captured by the limiting edge-ratio functions. In other words, the limiting edge-ratio relationships are local algebraic relationships mirroring the global vertical and rotational symmetries possessed by the underlying graph. Additionally, because row-reduction is local (in contrast to the combinatoric Laplacian which is global) the paper is able to introduce a mechanical verification method of proof for assertions about effective resistance identities.

(14) **Brad Isaacson**, NYC College of Technology (CUNY)
Email: brad.isaacson71@citytech.cuny.edu
Title: On a reciprocity formula for generalized Dedekind-Rademacher sums attached to three Dirichlet characters
Abstract: We define a three character analogue of the generalized Dedekind-Rademacher sum introduced by Hall, Wilson, and Zagier, and state its reciprocity formula, which contains all of the reciprocity formulas in the literature for generalized Dedekind-Rademacher sums attached (and not attached) to Dirichlet characters as special cases. We also review some of the generalized Dedekind-Rademacher sums in the literature to motivate our results.

(15) **Rauan Kaldybayev**, Williams College
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Title: Limiting behavior in missing sums of sumsets
Abstract: We study $|A + A|$ as a random variable, where $A \subseteq \{0, \ldots, N\}$ is a random subset such that each $0 \leq n \leq N$ is included with probability $0 < p < 1$, and where $A + A$ is the set of sums $a + b$ for a, b in A. Lazarev, Miller, and O'Bryant studied the distribution of $2N+1-|A+A|$, the number of summands not represented in $A+A$ when $p = 1/2$. A recent paper by Chu, King, Luntzlara, Martinez, Miller, Shao, Sun, and Xu generalizes this to all $p \in (0, 1)$, calculating the first and second moments of the number of missing summands and establishing exponential upper and lower bounds on the probability of missing exactly n summands, mostly working in the limit of large N. We provide exponential bounds on the probability of missing at least n summands, find another expression for the second moment of the number of missing summands, extract its leading-order behavior in the limit of small p, and show that the variance grows asymptotically slower than the mean, proving that for small p, the number of missing summands is very likely to be near its expected value.

(16) **Mizan Khan**, Eastern Connecticut State University
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Title: A conjecture for clean lattice parallelograms
Abstract: Let $P \subseteq \mathbb{R}^2$ be a convex lattice polygon containing at least one lattice point in its interior. The interior hull of P, denoted by $P^{(1)}$, is the convex closure of the set of lattice points in the interior of P, that is,

$$P^{(1)} = \operatorname{conv}\left(\operatorname{interior}(P) \cap \mathbb{Z}^2\right).$$

We can now form a finite nested sequence of interior hulls

$$P^{(1)} \supseteq P^{(2)} \supseteq P^{(3)} \supseteq \ldots,$$

where $P^{(2)}$ is the interior hull of $P^{(1)}$, $P^{(3)}$ is the interior hull of $P^{(2)}$, and so on.

We will present some experimental data supporting a conjecture on the average number of interior hulls for clean lattice parallelograms. (A lattice parallelogram is said to be clean if the only lattice points on its boundary are the 4 vertices.)

Joint work with Riaz Khan.

(17) **Nathaniel Kingsbury**, CUNY Graduate Center
Email: nkingsbury@gradcenter.cuny.edu
Title: The square-root law does not hold in the presence of zero divisors
Abstract: Let R be a finite ring and define the paraboloid $P = \{(x_1, \ldots, x_d) \in R^d | x_d = x_1^2 + \cdots + x_{d-1}^2\}$. Suppose that for a sequence of finite rings of size tending to infinity, the Fourier transform of P satisfies a square-root type bound constant C. Then all but finitely many of the rings are fields.

Most of our argument works in greater generality: let f be a polynomial with integer coefficients in $d - 1$ variables, with a fixed order of variable multiplications (so that it defines a function $R^{d-1} \to R$ even when R is noncommutative), and set $V_f = \{(x_1, \ldots, x_d) \in R^d | x_d = f(x_1, \ldots, x_{d-1})\}$.

If (for a sequence of finite rings of size tending to infinity) we have a square-root type bound on the Fourier transform of V_f, then all but finitely many of the rings are fields or matrix rings of small dimension.

(18) **Gergely Kiss**, Alfréd Rényi Institute of Mathematics, Budapest, Hungary
Email: kigergo57@gmail.com
Title: Solutions to the discrete Pompeiu problem and to the finite Steinhaus tiling problem
Abstract: Let K be a nonempty finite subset of the Euclidean space $\mathbb{R}^k$ ($k \geq 2$). In this talk we discuss the solution of the following so-called discrete Pompeiu problem. If a function $f\colon \mathbb{R}^k \to \mathbb{C}$ is such that the sum of f on every congruent copy of K is zero, then f vanishes everywhere. In fact, we solve a stronger, weighted version of this problem. As a corollary we obtain that every finite subset of $\mathbb{R}^k$ having at least two elements is a Jackson set; that is, no subset of $\mathbb{R}^k$ intersects every congruent copy of K in exactly one point.

Joint work with Miklós Laczkovich.

(19) **Sergei Konyagin**, Steklov Institute of Mathematics, Russia
Email: konyagin@mi-ras.ru
Title: On distinct angles in the plane
Abstract: The talk is based on our joint paper with Jonathan Passant and Misha Rudnev. We prove that if N points lie in convex position in the plane, then they determine $\gg N^{1+3/23+o(1)}$ distinct angles, provided no $N-1$ points lie on a common circle. This is the first super–linear bound on the distinct angle problem that has received recent attention.

(20) **Noah Kravitz**, Princeton University
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Title: Can you reconstruct a set from its subset sums?
Abstract: For a finite subset A of an abelian group, let $\mathrm{FS}(A)$ denote the multiset of its $2^{|A|}$ subset sums. Can you reconstruct A from $\mathrm{FS}(A)$? The answer in general is "no" (for instance, $\mathrm{FS}(\{1,3,-4\}) = \mathrm{FS}(\{-1,-3,4\})$), but in many cases, such as when the ambient group has no 2-torsion, we can obtain a combinatorial description of the fibers of FS.

Joint work with Federico Glaudo.

(21) **Andrzej Kukla**, Jagiellonian University in Kraków, Poland
Email: andrzej.kukla@student.uj.edu.pl
Title: Representing the number of binary partitions as sums of three squares
Abstract: Let $c_m(n)$ denote the number of partitions of n into parts that are powers of 2 such that part equal to 1 takes one among $2m$ colors and each part > 1 takes one among m colors. The study of this function was initiated in 2021 by Zmija and Ulas, who focused on its 2-adic behaviour. In particular, they found a formula for the 2-adic valuation of $c_m(n)$ that depends on the 2-adic valuation of m and the value of $t_n - t_{n-1}$, where t_n is the n-th term of the Prouhet-Thue-Morse sequence. During the talk we will be considering the diophantine equation $c_m(n) = x^2+y^2+z^2$. For fixed

m, we will characterize the set of natural numbers n, for which the solution does not exist, and then further investigate properties of these sets. The talk is based on an ongoing work on speaker's master's thesis.

(22) **Noah Lebowitz-Lockard**, University of Texas, Tyler, TX
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Title: On the smallest parts of partitions into distinct parts
Abstract: For a given integer n, let $D(n)$ be the set of partitions of n into distinct parts. Create a sum as follows. For each partition λ in $D(n)$, add the smallest element of λ if it is even and subtract it if it is odd. A classic theorem of Uchimura states that this quantity is equal to the number of divisors of n. We generalize this result to the sum of the kth smallest elements of partitions for a fixed value of k. We also consider some further generalizations, as well as variants for the smallest number not in a given partition.
Joint work with Rajat Gupta and Joseph Vandehey.

(23) **Paolo Leonetti**, Universitá degli Studi dell'Insubria, Italy
Email: leonettipaolo@gmail.com
Title: Most numbers are not normal
Abstract: Let S be the set of real numbers $x \in (0, 1]$ with the following property of being "strongly not normal": For all integers $b \geq 2$ and $k \geq 1$, the sequence of vectors made by the frequencies of all possible strings of length k in the b-adic representation of x has a maximal subset of accumulation points, and each of them is the limit of a subsequence with an index set of nonzero asymptotic density.

We show that S is a co-meager subset of $(0, 1]$, hence topologically large. Analogues are given in the context of regular matrices.

(24) **Jared Duker Lichtman**, Stanford University
Email: jared.d.lichtman@gmail.com
Title: Goldbach beyond the square-root barrier
Abstract: We show the primes have level of distribution 66/107 using triply well-factorable weights, and extend this level to 5/8 assuming Selberg's eigenvalue conjecture. This improves on the prior world record level of 3/5 by Maynard. As a result, we obtain new upper bounds for Goldbach representations of even numbers. This is the first use of a level of distribution beyond the 'square-root barrier' for the Goldbach problem, and leads to the greatest improvement on the problem since Bombieri-Davenport from 1966.

(25) **Qitong (George) Luan**, University of California, Los Angeles
Email: qluan21@g.ucla.edu
Title: On a pair of diophantine equations
Abstract: For relatively prime natural numbers a and b, we study the two equations $ax+by = (a-1)(b-1)/2$ and $ax+by+1 = (a-1)(b-1)/2$, which arise from the study of cyclotomic polynomials. Previous work showed that exactly one equation has a nonnegative integer solution, and the solution is unique. Our first result gives criteria to determine which equation is used

for a given pair (a,b). We then use the criteria to study the sequence of equations used by the pair $(a_n/\gcd(a_n,a_{n+1}), a_{n+1}/\gcd(a_n,a_{n+1}))$ from several special sequences $(a_n)_{n\geq 1}$, such as arithmetic progressions, geometric progressions and sequences satisfying Fibonacci-type recurrences. Furthermore, for each positive k, we construct a sequence $(a_n)_n$ whose consecutive terms use the two equations alternatively in groups of k. Lastly, we investigate the periodicity of the sequence of equations used by the pair $(k/\gcd(k,n), n/\gcd(k,n))$ as n increases.
Joint work with Sujith Uthsara Kalansuriya Arachchi, Hùng Việt Chu, Jiasen Liu, Rukshan Marasinghe, and Steven J. Miller.

(26) **Steve Miller**, Williams College
Email: sjm1@williams.edu
Title: The theory of normalization constants and Zeckendorf decompositions
Abstract: If we define the Fibonacci numbers to start 1, 2, 3, 5 and so on, we have a wonderful property: Every positive integer has a unique representation as a sum of non-adjacent terms. Called the Zeckendorf decomposition, we can prove many results about the summands, from the number in a typical decomposition converging to a Gaussian to the probabilities of gaps converging to a geometric decay. Many of these proofs are straightforward but tedious exercises in algebra. We present a new approach, which so far has just been applied to the distribution of gaps, but hopefully can work for related problems, which bypasses these calculations through the theory of normalization constants.

(27) **Piotr Miska**, Jagiellonian University in Kraków, Poland
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Title: On the Frobenius problem with restrictions on common divisors of coefficients
Abstract: Let m,s,t be positive integers with $t\leq s-2$ and let $a_1,a_2,\ldots,a_s$ be positive integers such that $(a_1,a_2,\ldots,a_{s-1})=1$. In the paper we prove that every sufficiently large positive integer can be written in the form $a_1\mu_1+a_2\mu_2+\ldots+a_s\mu_m$, where the positive integers $\mu_1,\mu_2,\ldots,\mu_s$ have no common divisor that is the m-th power of a positive integer greater than 1, but each t of the values $\mu_1,\mu_2,\ldots,\mu_s$ do have a common divisor that is the m-th power of a positive integer greater than 1. Moreover, we show that every sufficiently large positive integer can be written as a sum of positive integers $\mu_1,\mu_2,\ldots,\mu_s$ with no common divisor that is the m-th power of a positive integer greater than 1, but each $s-1$ of the values of $\mu_1,\mu_2,\ldots,\mu_s$ do have a common divisor that is the m-th power of a positive integer greater than 1.

Joint work with Maciej Zakarczemny (Cracow University of Technology).

(28) **Mel Nathanson**, Lehman College (CUNY)
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Title: Polynomial equations in infinitely many variables

(29) **Kevin O'Bryant**, College of Staten Island (CUNY)
Email: obryant@gmail.com
Title: Greedy B_h-sets
Abstract: A set X of integers is a B_h-set if every solution to $a_1 + \cdots + a_h = b_1 + \cdots + b_h$ with $a_i, b_i \in X$ has $\{a_1, \ldots, a_h\} = \{b_1, \ldots, b_h\}$ (as multisets). The main problem is to give inequalities connecting the cardinality and diameter of B_h-sets, and one obvious way to build thick B_h-sets is to be greedy. In this talk we survey old and new results on the greedy B_h-sets. The highlight of the new results is a nontrivial upper bound on the k-th element of the greedy B_h-set, provided that h is sufficiently large.

(30) **Carl Pomerance**, Dartmouth College
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Title: Matchable numbers
Abstract: For a natural number n let $D(n)$ denote the set of positive divisors of n and let $\tau(n) = \#D(n)$. Say n is *matchable* if there is a bijection from $D(n)$ to $\{1, 2, \ldots, \tau(n)\}$ with corresponding numbers relatively prime. For example, each number up to 7 is matchable, but 8 is not. This definition was made by Santos on MathOverflow in 2022; he asks if there are more matchable numbers than not. We prove this by showing the set of matchable numbers has an asymptotic density given by $\prod_{p\,\mathrm{prime}}(1 - 1/p^p) = .72199\ldots$.
This is joint work with Nathan McNew.

(31) **Firdavs Rakhmonov**, University of Rochester
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Title: The quotient set of the quadratic distance set over finite fields
Abstract: Let $\mathbb{F}_q^d$ be the d-dimensional vector space over the finite field $\mathbb{F}_q$ with q elements. For each non-zero r in $\mathbb{F}_q$ and $E \subset \mathbb{F}_q^d$, we define $W(r)$ as the number of quadruples $(x, y, z, w) \in E^4$ such that $Q(x-y)/Q(z-w) = r$, where Q is a non-degenerate quadratic form in d variables over $\mathbb{F}_q$. When $Q(\alpha) = \sum_{i=1}^d \alpha_i^2$ with $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{F}_q^d$, Pham (2022) recently used the machinery of group actions and proved that if $E \subset \mathbb{F}_q^2$ with $q \equiv 3 \pmod 4$ and $|E| \geq Cq$, then we have $W(r) \geq c|E|^4/q$ for any non-zero square number $r \in \mathbb{F}_q$, where C is a sufficiently large constant, c is some number between 0 and 1, and $|E|$ denotes the cardinality of the set E. I'll discuss the improvement and extension of Pham's result in two dimensions to arbitrary dimensions with general non-degenerate quadratic distances. As a corollary, we also generalize the sharp results on the Falconer type problem for the quotient set of distance set due to Iosevich-Koh-Parshall. Furthermore, we provide improved constants for the size conditions of the underlying sets. Joint work with Alex Iosevich and Doowon Koh.

(32) **James Sellers**, University of Minnesota Duluth
Email: jsellers@d.umn.edu
Title: Elementary proofs of congruences for POND and PEND partitions
Abstract: Recently, Ballantine and Welch considered two classes of integer partitions which they labeled POND and PEND partitions. These are integer partitions wherein the odd parts (respectively, the even parts) cannot be distinct. In recent work, I studied these two types of partitions from an arithmetic perspective and proved infinite families of mod 3 congruences satisfied by the two corresponding enumerating functions. I will talk about the generating functions for these enumerating functions, and I will also highlight the (induction) proofs that I utilized.

(33) **Steven Senger**, Missouri State University
Email: stevensenger@gmail.com
Title: Multi-parameter point-line incidence estimates in finite fields and applications
Abstract: We present some novel multi-parameter point-line incidence estimates in vector spaces over finite fields. These outperform the straightforward higher-dimensional analogs. We focus on an application to sums and products type problems.

(34) **I.D. Shkredov**, Purdue University
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Title: On universal sets and sumsets
Abstract: Let G be an abelian group. A set $A \subseteq G$ is called a $k-$*universal* set if for any $x_1, \dots, x_k \in G$ there exists $s \in G$ such that $x_1 + s, \dots, x_k + s \in A$. The term "universal set" was introduced by Alon, Bukh, and Sudakov in connection with the discrete Kakeya problem. We study the concept of universal sets from the additive–combinatorial point of view. Among other results we obtain some applications of this type of uniformity to sets avoiding solutions to linear equations, and get an optimal upper bound for the covering number of general sumsets.

(35) **Bartosz Sobolewski**, Jagiellonian University, Kraków, Poland and Montanuniversität Leoben, Austria
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Title: On block occurrences in the binary expansions of n and $n + t$
Abstract: Let $s(n)$ denote the sum of binary digits of a nonnegative integer n. In the recent years there has been significant progress concerning the behavior of the differences $s(n+t)-s(n)$, where t is a fixed nonnegative integer. In particular, Spiegelhofer and Wallner proved that for t having sufficiently many blocks 01 in its binary expansion, the set $\{n : s(n + t) \geq s(n)\}$ has natural density $> 1/2$ (partially confirming a conjecture by Cusick). Moreover, for such t the distribution $s(n+t) - s(n)$ is close to Gaussian. During the talk we consider an analogue of this problem concerning the function $r(n)$, which counts the occurrences of the block 11 in the binary expansion of n. In particular, we prove that the distribution of $r(n + t) - r(n)$ is

approximately Gaussian as well. We also discuss a generalization to an arbitrary block of binary digits.

Joint work with Lukas Spiegelhofer (Montanuniversität Leoben).

(36) **Gábor Somlai**, Eötvös Loránd University and Rényi Institute, Hungary
Email: zsomlei@gmail.com
Title: New method for old results of Rédei, Lovász and Schrijver
Abstract: Rédei proved that a set S of cardinality p in $\mathbb{F}_p^2$ determines at least $\frac{p+3}{2}$ directions or S is a line. We managed find a short proof for Rédei's result avoiding the theory of lacunary polynomials by proving the following statement. Let f be a polynomial over the finite field $\mathbb{F}_p$. Consider the elements of the range as integers in $\{0, 1, \ldots, p-1\}$. Assume that $\sum_{x \in \mathbb{F}_p} f(x) = p$. Then either $f = 1$ or $deg(f) \geq \frac{p-1}{2}$. The uniqueness (up to affine transformations) of the sets of size p in $\mathbb{F}_p^2$ was proved by Lovász and Schrijver. The same result follows from the almost uniqueness of the polynomials of degree $\frac{p-1}{2}$ of range sum p.

(37) **Christian Táfula**, Université de Montréal, Canada
Email: christian.tafula.santos@umontreal.ca
Title: Representation functions with prescribed rates of growth
Abstract: Let $h \geq 2$, and $b_1, \ldots, b_h$ be positive integers with gcd $= 1$. For a set $A \subseteq \mathbb{N}$, denote by $r_A(n)$ the number of solutions to the equation

$$b_1 k_1 + \ldots + b_h k_h = n$$

with $k_1, \ldots, k_h \in A$. For which functions F can we find A such that $r_A(n) \sim F(n)$? Or $r_A(n) \asymp F(n)$? In the asymptotic case, we show that for every F "regularly varying" in the range

$$\omega(\log x) = F(x) \leq \left(\tfrac{1}{(h-1)! b_1 \ldots b_h} + o(1)\right) x^{h-1}$$

(where $f = \omega(g)$ if $g = o(f)$), there is A such that $r_A(n) \sim F(n)$. In the order of magnitude case, there is A with $r_A(n) \asymp F(x)$ for every F non-decreasing such that $F(2x) \ll F(x)$ in the range $\log x \ll F(x) \ll x^{h-1}$. This extends earlier work of Erdős–Tetali and Vu, and addresses a question raised by Nathanson on which functions can be the r_A of some A.

If time allows, we will show for $1 \ll F(x) = o(\log x)$ a probabilistic heuristic for why there should not exist A with $r_A(n) \asymp F(n)$, agreeing with a conjecture of Erdős.

(38) **Marc Technau**, Paderborn University, Germany
Email: mtechnau@math.tugraz.at
Title: Cilleruelo's conjecture on the LCM of polynomial sequences
Abstract: We discuss a conjecture of Cilleruelo on the growth of the least common multiple of consecutive values of a polynomial and subsequent progress towards it in work of Maynard–Rudnick and Sah. In recent work, the speaker and, independently, Alexei Entin made further advances by exploiting symmetries amongst the roots of the polynomials in question. We shall discuss these approaches and related beautiful work of Baier and Dey.

(39) **Fred Tyrell**, University of Bristol, UK
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Title: New lower bounds for cap sets
Abstract: A cap set is a subset of $\mathbb{F}_3^n$ with no solutions to $x+y+z=0$ other than when $x=y=z$. The cap set problem asks how large a cap set can be, and is an important problem in additive combinatorics and combinatorial number theory. In this talk, I will introduce the problem, give some background and motivation, and describe how I was able to provide the first progress in 20 years on the lower bound for the size of a maximal cap set. Building on a construction of Edel, we use improved computational methods and new theoretical ideas to show that, for large enough n, there is always a cap set in $\mathbb{F}_3^n$ of size at least 2.218^n. I will then also discuss recent developments, including an extension of this result by Google DeepMind.

(40) **Maciej Ulas**, Jagiellonian University, Kraków, Poland
Email: maciej.ulas@gmail.com
Title: On the Diophantine equation $x^3 \pm y^3 = a^k \pm b^k$
Abstract: We consider the Diophantine equation $x^3 \pm y^3 = a^k \pm b^k$ from both the theoretical as well as the experimental point of view. In particular, we prove that for $k = 4, 6$, regardless of the signs chosen, our equation has infinitely many co-prime positive integer solutions. This confirms expectations outlined by Wagstaff. Additionally, for $k = 5, 7$ we report on computations of all co-prime positive integer solutions (x, y, a, b) satisfying the condition $\max\{a, b\} \le 50000$.

(41) **Aleksei Volostnov**, Moscow Institute of Physics and Technology, Russia
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Title: On the additive energy of roots
Abstract: Let p be a prime number, $f \in \mathbb{F}_p[x]$ be a polynomial of small degree and a set $A \subset \mathbb{F}_p$ have sufficiently small cardinality in terms of p. We study the number of solutions to the equation (in $\mathbb{F}_p$)

$$x_1 + x_2 = x_3 + x_4, \quad f(x_1), f(x_2), f(x_3), f(x_4) \in A,$$

provided that A has small doubling. Namely, we improve the upper bound from recent work by B. Kerr, I. D. Shkredov, I. E. Shparlinski and A. Zaharescu.

Moreover, we address questions of cardinalities $|A+A|$ vs $|f(A)+f(A)|$. In particular, we prove that

$$\max(|A+A|, |A^3+A^3|) \gg |A|^{16/15}$$
$$\max(|A+A|, |A^4+A^4|) \gg |A|^{25/24}$$
$$\max(|A+A|, |A^5+A^5|) \gg |A|^{25/24}.$$

(42) **Trevor D. Wooley**, Purdue University
Email: twooley@purdue.edu
Title: Unrepresentation theory and sums of powers
Abstract: We report on recent and on-going work joint with Jörg Brüdern concerning problems involving the representation of integer sequences by sums of powers. Our new tool is an upper bound for moments of smooth Weyl sums restricted to major arcs. This permits progress to be made on Waring's problem and other problems involving mixed sums of powers and primes. We will focus on recent progress concerning unrepresentation theory (bounds for exceptional sets).