CANT 2024

.

Twenty-second Annual Workshop on Combinatorial and Additive Number Theory

CUNY Graduate Center

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Abstracts (preliminary list)

(1) **Sukumar Das Adhikari**, Ramakrishna Mission Vivekananda Educational and Research Institute (RKMVERI), India
Email: adhikarisukumar@gmail.com
Title: Some elementary algebraic and combinatorial methods in the study of zero-sum theorems
Abstract: Originating from a beautiful theorem of Erdos-Ginzberg-Ziv about sixty years ago and some other questions asked around the same time, the area of zero-sum theorems has many interesting results and several unanswered questions.

Several authors have introduced interesting elementary algebraic techniques to deal with these problems. We describe some experiments with these elementary algebraic methods and some combinatorial ones, in a weighted generalization in the area of Zero-sum Combinatorics.

(2) **Adrian Beker**, University of Zagreb, Croatia
Email: adrian.beker@math.hr
Title: On a problem of Erdős and Graham about consecutive sums in strictly increasing sequences
Abstract: Given a finite sequence of integers $a = (a_i)_{1 \leq i \leq k}$, let $S(a)$ denote the set of its consecutive sums, that is, sums of the form $\sum_{i=u}^{v} a_i$ with $1 \leq u \leq v \leq k$. Erdős and Graham asked whether there exists a constant $c > 0$ such that, for all positive integers n, there is such a sequence in $\{1, \ldots, n\}$ which is strictly increasing and satisfies $|S(a)| \geq cn^2$.

The obvious candidate consisting of all integers from 1 up to n falls short of having this property due to reasons related to the multiplication table problem. On the other hand, if we drop the monotonicity assumption, such sequences were shown to exist by Hegyvári via a construction based on Sidon sets. In this talk, I will present two constructions, one probabilistic and the other deterministic, that give an affirmative answer to the starting question. I will also discuss some non-trivial upper bounds on the size of $S(a)$ in this setting.

(3) **Christine K. Chang**, CUNY Graduate Center
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Title: Hybrid statistics of the maxima of a random model of the zeta function over short intervals
Abstract: We will present a matching upper and lower bound for the right

tail probability of the maximum of a random model of the Riemann zeta function over short intervals. In particular, we show that the right tail interpolates between that of log-correlated and IID random variables as the interval varies in length. We will also discuss a new normalization for the moments over short intervals. This result follows the recent work of Arguin-Dubach-Hartung and is inspired by a conjecture by Fyodorov-Hiary-Keating on the local maximum over short intervals.

(4) **Eric Dolores Cuenca**, Pusan National University, Korea
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Title: Zeta values as an algebra over an operad
Abstract: Denote the operad of finite posets by FP. In number theory, the field of rational zeta series studies series of the form $\sum_{i=1}^{\infty} a_i(\zeta(i+1)-1), a_i \in \mathbb{Q} \forall i \in \mathbb{N}$, where $\zeta(k)$ is the Riemann zeta function $\zeta(k) = \sum_{n=1}^{\infty} \frac{1}{n^k}$. By studying zeta values as algebras over the operad of posets, we show the following identity, for $a > 1, a \in \mathbb{N}$:

$$\sum_{n=i}^{\infty}(-1)^{n+1}\binom{n}{i}\zeta(n+1,a) = (-1)^{i+1}\zeta(i+1,a+1),$$

here, $\zeta(k,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^k}$ is the Hurwitz zeta function.

On January 2023 we put the left side of the identity on several private software, but none of them produced any output. We presented our work in the Wolfram Technology Conference 2014, where their team kindly verified that the left side of the identity is equal to the right side of the identity.
Joint work with Jose Mendoza-Cortes, Michigan State University

(5) **Jin-Hui Fang**, Nanjing Normal University, China
Email: fangjinhui1114@163.com
Title: On Cilleruelo-Nathanson's method in Sidon sets
Abstract: For nonnegative integers h, g with $h \geq 2$, a set $\mathcal{A}$ of nonnegative integers is defined as a $B_h[g]$ sequence if, for every nonnegative integer n, the number of representations of n with the form $n = a_1 + a_2 + \cdots + a_h$ is no larger than g, where $a_1 \leq \cdots \leq a_h$ and $a_i \in \mathcal{A}$ for $i = 1, 2, \cdots, h$. Let $\mathbb{Z}$ be the set of integers and $\mathbb{N}$ be the set of positive integers. In 2013, by introducing the method of *Inserting Zeros Transformation*, Cilleruelo and Nathanson obtained the following nice result: let $f : \mathbb{Z} \to \mathbb{N} \bigcup \{0, \infty\}$ be any function such that $\liminf_{|n|\to\infty} f(n) \geq g$ and let $\mathcal{B}$ be any $B_h[g]$ sequence. Then, for any decreasing function $\epsilon(x) \to 0$ as $x \to \infty$, there exists a sequence $\mathcal{A}$ of integers such that $r_{\mathcal{A},h}(n) = f(n)$ for all $n \in \mathbb{Z}$ and $\mathcal{A}(x) \gg B(x\epsilon(x))$. In 2022, Nathanson further considered Sidon sets for linear forms. Recently, we apply the Inserting Zeros Transformation into Sidon sets for linear forms and generalize the above result related to the inverse problem of representation functions.

(6) **Daniel Flores**, Purdue University
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Title: Existence of infinitely non-trivial prime K-multimagic squares

Abstract: Given $N \in \mathbb{N}$ we say that a matrix $\mathbf{Z} = (z_{i,j})_{1\le i,j\le N} \in \mathbf{Z}^{N\times N}$ is a magic square of order N if the sum of the entries of its rows, columns, and two main diagonals are equal. Let $\mathbf{Z}^{\circ k} = (z_{i,j}^k)_{1\le i,j\le N}$ denote the kth Hadamard power of $\mathbf{Z}$. Then we say that a matrix $\mathbf{Z}$ is a K-multimagic square if for all $1 \le k \le K$ the matrix $\mathbf{Z}^{\circ k}$ is a magic square.

Here we show that given $K \ge 2$ and $N > 1 + 2K(K+1)$ there exist infinitely many non-trivial K-multimagic squares of order N in which all the entries are prime. We accomplish this via an application of the Hardy-Littlewood circle method and the Green-Tao theorem.

(7) **N. Hegyvári**, Eötvös Loránd University and Rényi Institute, Hungary
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Title: On the structures of sets in $\mathbb{N}^k$ having thin subset sums
Abstract: For any $X \subseteq \mathbb{N}^k$ let

$$FS(X) := \{\sum_{i=1}^{\infty} \varepsilon_i x_i :\ x_i \in X,\ \varepsilon_i \in \{0,1\},\ \sum_{i=1}^{\infty} \varepsilon_i < \infty\}$$

Erdős called a sequence $A \subseteq \mathbb{N}$ *complete* if every sufficiently large number belongs to $FS(A)$. In a higher dimension too, the necessary condition that the subset sums of a subset $X \subseteq \mathbb{N}^k$ represent all far points of $\mathbb{N}^k$ should be the condition $X(N) > k\log_2 N - t_X$ for some t_X, i.e. X is complete respect to the region $R = \{x = (x_1, x_2, \dots, x_k) : x_i \ge r_i\}$, $r_i \in \mathbb{N}$, $i = 1, 2, \dots, k$.

Let $A = \{a_1 < a_2 < \dots < a_n < \dots\}$ be an infinite sequence of integers. We say that A is *weakly thin* if $\limsup_{n\to\infty} \frac{\log a_n}{\log n} = \infty$, or equivalently $A(n) := \sum_{a_i \le n} 1 = n^{g(n)}$, where $A(n)$ is the counting function of A and $\liminf_{n\to\infty} g(n) = 0$. A set $B \subseteq \mathbb{N}$ is said to be *thick* if it is not weakly thin. Let $X \subseteq \mathbb{N}^k$. X is said to be *thin complete set* respect to R if $X(N) > k\log_2 R(N) - t_X$ for some t_X and $FS(X) \supseteq R$. We prove that if $R = z + \mathbb{N}^k$ and A is complete with respect to R, then all projections of A onto for all axis f_i are thick. We also determine regions for which there exists thin complete sets. Some related results are also discussed.

This is joint work with Máté Pálfy and Erfei Yue.

(8) **Russell Jay Hendel**, Towson University
Email: RHendel@Towson.edu
Title: Local distance-resistance functions equivalent to global symmetries in electric circuit families
Abstract: In a recent paper Hendel explored the computational attributes of an algorithm introduced by Barrett, Evans, and Francis, which, among other things, studied distance resistance in a family of circuits whose underlying graphs consisted of n rows of upright equilateral triangles (n-grids). Two important conjectures supported by numerical evidence were presented: one related to the asymptotic behavior of iterated use of the algorithm on an initial n grid as n goes to infinity. The second conjecture showed that as n grows large certain limiting ratios emerge among specified edges in the circuits resulting from a large number of repeated applications of the algorithm to an initial n-grid. The purpose of this paper is to provide insight into these asymptotic or limiting edge ratios. After introducing

the algorithm and reviewing the original conjectures, the main part of this paper studies a family of n-grids whose edge labels are determined using these limiting edge-ratios functions. The main result proven is that these n-grids as well as the graphs derived from repeated application of the algorithm possess vertical and rotational symmetries and also continue to satisfy the relationships captured by the limiting edge-ratio functions. In other words, the limiting edge-ratio relationships are local algebraic relationships mirroring the global vertical and rotational symmetries possessed by the underlying graph. Additionally, because row-reduction is local (in contrast to the combinatoric Laplacian which is global) the paper is able to introduce a mechanical verification method of proof for assertions about effective resistance identities.

(9) **Rauan Kaldybayev**, Williams College
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Title: Limiting behavior in missing sums of sumsets
Abstract: We study $|A + A|$ as a random variable, where $A \subseteq \{0, \ldots, N\}$ is a random subset such that each $0 \leq n \leq N$ is included with probability $0 < p < 1$, and where $A + A$ is the set of sums $a + b$ for a, b in A. Lazarev, Miller, and O'Bryant studied the distribution of $2N+1-|A+A|$, the number of summands not represented in $A+A$ when $p = 1/2$. A recent paper by Chu, King, Luntzlara, Martinez, Miller, Shao, Sun, and Xu generalizes this to all $p \in (0, 1)$, calculating the first and second moments of the number of missing summands and establishing exponential upper and lower bounds on the probability of missing exactly n summands, mostly working in the limit of large N. We provide exponential bounds on the probability of missing at least n summands, find another expression for the second moment of the number of missing summands, extract its leading-order behavior in the limit of small p, and show that the variance grows asymptotically slower than the mean, proving that for small p, the number of missing summands is very likely to be near its expected value.

(10) **Gergely Kiss**, Alfréd Rényi Institute of Mathematics, Budapest, Hungary
Email: kigergo57@gmail.com
Title: Solutions to the discrete Pompeiu problem and to the finite Steinhaus tiling problem
Abstract: Let K be a nonempty finite subset of the Euclidean space $\mathbb{R}^k$ ($k \geq 2$). In this talk we discuss the solution of the following so-called discrete Pompeiu problem. If a function $f : \mathbb{R}^k \to \mathbb{C}$ is such that the sum of f on every congruent copy of K is zero, then f vanishes everywhere. In fact, we solve a stronger, weighted version of this problem. As a corollary we obtain that every finite subset of $\mathbb{R}^k$ having at least two elements is a Jackson set; that is, no subset of $\mathbb{R}^k$ intersects every congruent copy of K in exactly one point.

This is a joint work with Miklós Laczkovich.

(11) **Sergei Konyagin**. Stekhlov Institute, Russia
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Title: tba

(12) **Noah Lebowitz-Lockard**, University of Texas, Tyler, TX
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Title: On the smallest parts of partitions into distinct parts
Abstract: For a given integer n, let $D(n)$ be the set of partitions of n into distinct parts. Create a sum as follows. For each partition λ in $D(n)$, add the smallest element of λ if it is even and subtract it if it is odd. A classic theorem of Uchimura states that this quantity is equal to the number of divisors of n. We generalize this result to the sum of the kth smallest elements of partitions for a fixed value of k. We also consider some further generalizations, as well as variants for the smallest number not in a given partition.
Joint work with Rajat Gupta and Joseph Vandehey.

(13) **Paolo Leonetti**, Universitá degli Studi dell'Insubria, Italy
Email: leonettipaolo@gmail.com
Title: Most numbers are not normal
Abstract: Let S be the set of real numbers $x \in (0,1]$ with the following property of being "strongly not normal": For all integers $b \geq 2$ and $k \geq 1$, the sequence of vectors made by the frequencies of all possible strings of length k in the b-adic representation of x has a maximal subset of accumulation points, and each of them is the limit of a subsequence with an index set of nonzero asymptotic density.

We show that S is a co-meager subset of $(0,1]$, hence topologically large. Analogues are given in the context of regular matrices.

(14) **Qitong (George) Luan**, University of California, Los Angeles
Email: qluan21@g.ucla.edu
Title: On a pair of diophantine equations
Abstract: For relatively prime natural numbers a and b, we study the two equations $ax+by = (a-1)(b-1)/2$ and $ax+by+1 = (a-1)(b-1)/2$, which arise from the study of cyclotomic polynomials. Previous work showed that exactly one equation has a nonnegative integer solution, and the solution is unique. Our first result gives criteria to determine which equation is used for a given pair (a,b). We then use the criteria to study the sequence of equations used by the pair $(a_n/\gcd(a_n,a_{n+1}), a_{n+1}/\gcd(a_n,a_{n+1}))$ from several special sequences $(a_n)_{n\geq 1}$, such as arithmetic progressions, geometric progressions and sequences satisfying Fibonacci-type recurrences. Furthermore, for each positive k, we construct a sequence $(a_n)_n$ whose consecutive terms use the two equations alternatively in groups of k. Lastly, we investigate the periodicity of the sequence of equations used by the pair $(k/\gcd(k,n), n/\gcd(k,n))$ as n increases.
Joint work with Sujith Uthsara Kalansuriya Arachchi, Hùng Việt Chu, Jiasen Liu, Rukshan Marasinghe, and Steven J. Miller.

(15) **Steve Miller**, Williams College
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Title: The theory of normalization constants and Zeckendorf decompositions
Abstract: If we define the Fibonacci numbers to start 1, 2, 3, 5 and so on, we have a wonderful property: Every positive integer has a unique representation as a sum of non-adjacent terms. Called the Zeckendorf decomposition, we can prove many results about the summands, from the number in a typical decomposition converging to a Gaussian to the probabilities of gaps converging to a geometric decay. Many of these proofs are straightforward but tedious exercises in algebra. We present a new approach, which so far has just been applied to the distribution of gaps, but hopefully can work for related problems, which bypasses these calculations through the theory of normalization constants.

(16) **Mel Nathanson**, CUNY
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Title: tba

(17) **Firdavs Rakhmonov**, University of Rochester
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Title: The quotient set of the quadratic distance set over finite fields
Abstract: Let $\mathbb{F}_q^d$ be the d-dimensional vector space over the finite field $\mathbb{F}_q$ with q elements. For each non-zero r in $\mathbb{F}_q$ and $E \subset \mathbb{F}_q^d$, we define $W(r)$ as the number of quadruples $(x, y, z, w) \in E^4$ such that $Q(x-y)/Q(z-w) = r$, where Q is a non-degenerate quadratic form in d variables over $\mathbb{F}_q$. When $Q(\alpha) = \sum_{i=1}^{d} \alpha_i^2$ with $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{F}_q^d$, Pham (2022) recently used the machinery of group actions and proved that if $E \subset \mathbb{F}_q^2$ with $q \equiv 3 \pmod 4$ and $|E| \geq Cq$, then we have $W(r) \geq c|E|^4/q$ for any non-zero square number $r \in \mathbb{F}_q$, where C is a sufficiently large constant, c is some number between 0 and 1, and $|E|$ denotes the cardinality of the set E.

In this talk, I'll discuss the improvement and extension of Pham's result in two dimensions to arbitrary dimensions with general non-degenerate quadratic distances. As a corollary of our results, we also generalize the sharp results on the Falconer type problem for the quotient set of distance set due to Iosevich-Koh-Parshall. Furthermore, we provide improved constants for the size conditions of the underlying sets.

This is joint work with Alex Iosevich and Doowon Koh.

(18) **James Sellers**, University of Minnesota Duluth
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Title: Elementary proofs of congruences for POND and PEND partitions
Abstract: Recently, Ballantine and Welch considered two classes of integer partitions which they labeled POND and PEND partitions. These are integer partitions wherein the odd parts (respectively, the even parts) cannot be distinct. In recent work, I studied these two types of partitions from an arithmetic perspective and proved infinite families of mod 3 congruences

satisfied by the two corresponding enumerating functions. I will talk about the generating functions for these enumerating functions, and I will also highlight the (induction) proofs that I utilized.

(19) **Gábor Somlai**, Eötvös Loránd University and Rényi Institute, Hungary
Email: zsomlei@gmail.com
Title: New method for old results of Rédei, Lovász and Schrijver
Abstract: Rédei proved that a set S of cardinality p in $\mathbb{F}_p^2$ determines at least $\frac{p+3}{2}$ directions or S is a line.

We managed find a short proof for Rédei's result avoiding the theory of lacunary polynomials by proving the following statement. Let f be a polynomial over the finite field $\mathbb{F}_p$. Consider the elements of the range as integers in $\{0, 1, \dots, p-1\}$. Assume that $\sum_{x \in \mathbb{F}_p} f(x) = p$. Then either $f = 1$ or $deg(f) \geq \frac{p-1}{2}$.

The uniqueness (up to affine transformations) of the sets of size p in $\mathbb{F}_p^2$ was proved by Lovász and Schrijver. The same result follows from the almost uniqueness of the polynomials of degree $\frac{p-1}{2}$ of range sum p.

(20) **Christian Táfula**, Université de Montréal, Canada
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Title: Representation functions with prescribed rates of growth
Abstract: Let $h \geq 2$, and $b_1, \dots, b_h$ be positive integers with gcd $= 1$. For a set $A \subseteq \mathbb{N}$, denote by $r_A(n)$ the number of solutions to the equation

$$b_1 k_1 + \dots + b_h k_h = n$$

with $k_1, \dots, k_h \in A$. For which functions F can we find A such that $r_A(n) \sim F(n)$? Or $r_A(n) \asymp F(n)$? In the asymptotic case, we show that for every F "regularly varying" in the range

$$\omega(\log x) = F(x) \leq \left(\tfrac{1}{(h-1)! b_1 \dots b_h} + o(1)\right) x^{h-1}$$

(where $f = \omega(g)$ if $g = o(f)$), there is A such that $r_A(n) \sim F(n)$. In the order of magnitude case, there is A with $r_A(n) \asymp F(x)$ for every F non-decreasing such that $F(2x) \ll F(x)$ in the range $\log x \ll F(x) \ll x^{h-1}$. This extends earlier work of Erdős–Tetali and Vu, and addresses a question raised by Nathanson on which functions can be the r_A of some A.

If time allows, we will show for $1 \ll F(x) = o(\log x)$ a probabilistic heuristic for why there should not exist A with $r_A(n) \asymp F(n)$, agreeing with a conjecture of Erdős.

(21) **Maciej Ulas**, Jagiellonian University, Kraków, Poland
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Title: On the Diophantine equation $x^3 \pm y^3 = a^k \pm b^k$
Abstract: We consider the Diophantine equation $x^3 \pm y^3 = a^k \pm b^k$ from both the theoretical as well as the experimental point of view. In particular, we prove that for $k = 4, 6$, regardless of the signs chosen, our equation has infinitely many co-prime positive integer solutions. This confirms expectations outlined by Wagstaff. Additionally, for $k = 5, 7$ we report on computations of all co-prime positive integer solutions (x, y, a, b) satisfying

the condition $\max\{a, b\} \leq 50000$.