CANT 2025

.

Twenty-third Annual Workshop on Combinatorial and Additive Number Theory

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Abstracts

(1) **Carlo Francisco E. Adajar**, University of Georgia
Email: paco@uga.edu
Title: On the distribution of $v_p(\sigma(n))$
Abstract: For a positive integer m and a prime p, we write $\sigma(m) := \sum_{d|m} d$ for the sum of the divisors of m, and $v_p(m) := \max\{k \in \mathbf{Z}_{\geq 0} : p^k \mid m\}$ for the p-adic valuation of m, i.e., the exponent of p in the prime factorization of m. For each prime p, we give an asymptotic expression for the count

$$\#\{n \leq x : v_p(\sigma(n)) = k\}$$

as $x \to \infty$, uniformly for $k \ll \log\log x$. We then deduce an asymptotic for the count of $n \leq x$ such that $v_p(\sigma(n)) < v_p(n)$ as $x \to \infty$.
This talk is based on ongoing work with Paul Pollack.

(2) **Samuel Allen Alexander**, U.S. Securities and Exchange Commission
Title: Hindman's theorem and the hyperreals
Email: samuelallenalexander@gmail.com
Abstract: Hindman's theorem says that if the natural numbers are colored using finitely many colors, then there exists some color c and some infinite $S \subseteq \mathbb{N}$ such that for every finite nonempty subset $\{n_1, \ldots, n_k\}$ of S, $n_1 + \cdots + n_k$ is color c. We present a proof using hyperreal numbers, and a stronger version of the theorem involving hyperreal numbers.
Some of this material was previously published in 2024 in the Journal of Logic and Analysis.

(3) **Daniel Baczkowski**, University of Findlay
Email: baczkowski@findlay.edu
Title: Diophantine equations involving arithmetic functions and factorials
Abstract: F. Luca proved for any fixed rational number $\alpha > 0$ that the Diophantine equations $\alpha\, m! = f(n!)$, where f is either the Euler function, the divisor sum function, or the function counting the number of divisors, have finitely many integer solutions in m and n. In joint work with Novaković we generalize the mentioned result and show that Diophantine equations of the form $\alpha\, m_1! \cdots m_r! = f(n!)$ have finitely many integer solutions, too. In addition, we do so by including the case f is the sum of k^{th} powers of divisors function. Moreover, the same holds by replacing some of the factorials with certain examples of Bhargava factorials.

(4) **Paul Baginski**, Fairfield University
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Title: The plus-minus Davenport constant of some finite abelian groups
Abstract: Let G be a finite abelian group, written additively. A sequence $S = \{g_1, \ldots, g_n\}$ of not necessarily distinct elements of G is a zero-sum sequence if $\sum_{i=1}^{n} g_i = 0$. The (classical) Davenport constant $D(G)$ is the least number n such that every sequence S of length n or greater has a zero-sum subsequence. The Davenport constant has long been studied due to its applications to algebraic number theory and other branches of mathematics. Many variations of the Davenport constant have been proposed, including one known as the plus-minus Davenport constant $D_{\pm}(G)$, which has applications to combinatorics. Specifically, a sequence $S = \{g_1, \ldots, g_n\}$ of not necessarily distinct elements of G is a plus-minus zero-sum sequence if there are $\lambda_i \in \{1, -1\}$ such that $\sum_{i=1}^{n} \lambda_i g_i = 0$. The plus-minus Davenport constant is the least number n such that every sequence S of length n or greater has a plus-minus zero-sum subsequence. In this talk, we will give an update on the current knowledge about the plus-minus Davenport constant $D_{\pm}(G)$ for several finite abelian groups G.

(5) **Gautami Bhowmik**, Université Lille, France
Email: gautami.bhowmik@univ-lille.fr
Title: On the Telhcirid problem
Abstract: We consider the digital reverse of integers, in particular those of primes. A palindromic prime number is a popular example of a prime whose reverse is also a prime and the infinitude of such primes is one among the open conjectures in the area. We will discuss reversed primes in arithmetic progression built on ideas of Mauduit-Rivat and Maynard.
This is joint work with Yuta Suzuki.

(6) **Jörg Brüdern**, Universität Göttingen, Germany
Email: jbruede@gwdg.de
Title: Expander estimates for cubes
Abstract: Suppose that $\mathcal{A}$ is a subset of the natural numbers. The supremum α of all t with

$$\limsup N^{-t} \#\{a \in \mathcal{A} : a \leq N\} > 0$$

is the *exponential density* of $\mathcal{A}$.

We examine what happens if one adds a power to $\mathcal{A}$. Fix $k \geq 2$, and let β_k be the exponential density of

$$\{x^k + a : x \in \mathbb{N},\, a \in \mathcal{A}\}.$$

It is easy to see that $\beta_2 = \min(1, \frac{1}{2} + \alpha)$. One might guess that

$$\beta_k = \min(1, \frac{1}{k} + \alpha) \qquad (*)$$

holds for all k, but we are far from a proof. All current world records for this problem are due to Davenport, and are 80 years old. In this interim report on ongoing work with Simon Myerson, we describe a method for $k = 3$ that improves Davenport's results when $\alpha > 3/5$, and that confirms (*) in an interval $(\alpha_0, 1]$. A concrete value for α_0 will be released during the talk, and if time permits, we also discuss the perspectives to generalize the approach to larger values of k.

(7) **Jonathan Chapman**, University of Warwick, UK
Email: Jonathan.Chapman@warwick.ac.uk
Title: Counting commuting integer matrices
Abstract: Consider the set of pairs of $d \times d$ matrices (A, B) whose entries are all integers with absolute value at most N. We call (A, B) a *commuting pair* if $AB = BA$. Browning, Sawin, and Wang recently showed that the number of commuting pairs is at most $O_d(N^{d^2+2-\frac{2}{d+1}})$. They further conjectured that the lower bound $\Omega_d(N^{d^2+1})$, which comes from letting A or B be a multiple of the identity matrix, should be sharp. In this talk, I will discuss progress on the cases $d = 2$ and $d = 3$, where we show that this conjecture holds. I will also demonstrate how our approach relates counting commuting pairs of matrices to the study of restricted divisor correlations in number theory.
Joint work with Akshat Mudgal (University of Warwick).

(8) **Scott Chapman**, Sam Houston State University
Email: stc008@shsu.edu
Title: Betti elements and non-unique factorizations
Abstract: Let M be a commutative cancellative reduced atomic monoid with set of atoms (or irreducibles) $\mathcal{A}(M)$. Given a nonunit x in M, let $Z(x)$ represent the set of factorizations of x into atoms. Define a graph ∇_x whose vertex set is $Z(x)$ where two vertices are joined by an edge if these factorizations share an atom. Call x a *Betti element* of M if the graph ∇_x is disconnected. Betti elements have proven to be a powerful tool in the study of nonunique factorizations of elements in monoids. In particular, over the past several years many papers have used Betti elements to study factorizaton properties in *affine monoids* (i.e., finitely generated additive submonoids of $\mathbb{N}_0^k$ for some positive integer k). Several strong results have been obtained when M is a numerical monoid (i.e., $k = 1$ above). In this talk, we will review the basic properties of Betti elements and some of the results regarding affine monoids mentioned above. We will then extend this study to more general rings and monoids which are commutative and cancellative. We focus on two cases: (I) when the monoid M has a single Betti element, (II) when each atom of M divides every Betti element. We call those monoids satisfying condition (II) as having *full atomic support*. We show using elementary arguments that a monoid of type (I) is actually of full atomic support. We close by showing for a monoid of full atomic support that the catenary degree, the tame degree, and the omega primality constant (three well studied invariants in the nonunique factorization literature) can be easily computed from the monoid's set of Betti elements.

(9) **Shashi Chourasiya**, University of New South Wales, Australia
Email: s.chourasiya@unsw.edu.au
Title: Power-free palindromes and reversed primes
Abstract: Several long-standing conjectures in number theory are related

to the digital properties of integers. Historically, such problems have been confined to the realm of elementary number theory, but recently huge breakthroughs have been made by applying deep analytical techniques. In this talk, we discuss some very recent results on this topic, focusing on palindromes and reversed primes. We first establish that for all bases $b \geq 26000$, there exist infinitely many prime numbers p for which $\{\overleftarrow{p}\}$ is square-free. Furthermore, we demonstrate the existence of infinitely many palindromes (with $n = \overleftarrow{n}$) that are cube-free.
This is based on joint work with Daniel R. Johnston.

(10) **Hopper Clark**, Bates College
Email: hclark2@bates.edu
Title : Patterns among Ulam words
Abstract: In 1964, Stanislaw Ulam wrote about the Ulam sequence: beginning with 1 and 2, the next term is the smallest unique sum of two different earlier terms. In 2020, the parallel notion of the set of Ulam words, $\mathcal{U}$, was introduced by Bade, Cui, Labelle, and Li, which looks at concatenations of words in F_2, the free group on two generators. In this talk, we will discuss patterns of words in $\mathcal{U}$, touching on both proven results and conjectured ones. We will see how these patterns come to life visually, and see how they produce images such as the discrete Sierpínski triangle.

(11) **Taylor Daniels**, Purdue University
Email: daniel84@purdue.edu
Title: Vanishing Legendre-17-signed partition numbers
Abstract: For odd primes p let $\chi_p(r) := (\frac{r}{p})$ denote the Legendre symbol. With this, the Legendre-signed partition numbers, denoted $\mathfrak{p}(n, \chi_p)$, are then defined to be the coefficients appearing in the series expansion

$$\prod_{r=1}^{p-1} \prod_{m=0}^{\infty} \frac{1}{1 - \chi_p(r) q^{mp+r}} = 1 + \sum_{n=1}^{\infty} \mathfrak{p}(n, \chi_p) q^n.$$

It is known that: (1) one has $\mathfrak{p}(n, \chi_5) = 0$ for all $n \equiv 2 \pmod{10}$; and (2) the sequences $(\mathfrak{p}(n, \chi_p))_{n \geq 1}$ do not have such a periodic vanishing whenever $p \not\equiv 1 \pmod 8$ and $p \neq 5$. In this talk we discuss the recent result that $\mathfrak{p}(n, \chi_{17})$ vanishes only when the input n is odd and $1 - 24n$ is congruent to a quartic residue (mod 17), as well as a similar vanishing in the sequence $(\mathfrak{p}(n, -\chi_{17}))_{n \geq 1}$.

(12) **Aritram Dhar**, University of Florida
Email: aritramdhar@ufl.edu
Title: A bijective proof of an identity of Berkovich and Uncu
Abstract: The BG-rank BG(π) of an integer partition π is defined as

$$\mathrm{BG}(\pi) := i - j$$

where i is the number of odd-indexed odd parts and j is the number of even-indexed odd parts of π. In a recent work, Fu and Tang ask for a direct combinatorial proof of the following identity of Berkovich and Uncu

$$B_{2N+\nu}(k,q) = q^{2k^2-k} \begin{bmatrix} 2N+\nu \\ N+k \end{bmatrix}_{q^2}$$

for any integer k and non-negative integer N where $\nu \in \{0,1\}$, $B_N(k,q)$ is the generating function for partitions into distinct parts less than or equal to N with BG-rank equal to k and $\begin{bmatrix} a+b \\ b \end{bmatrix}_q$ is a Gaussian binomial coefficient. In this talk, I will give a bijective proof of Berkovich and Uncu's identity along the lines of Vandervelde and Fu and Tang's idea.
This is joint work with Avi Mukhopadhyay.

(13) **Robert W. Donley, Jr.**, CUNY and Institute for Advanced Study
Email: rdonley@qcc.cuny.edu
Title: A classification for 1-factorizations of small order
Abstract: A graph G admits a 1-factorization if its edge set decomposes into disjoint perfect matchings. When G is bipartite, the equivalency classes of such graphs are determined by orbits of 0/1-semi-magic squares under row and column permutations. By the Birkhoff-von Neumann theorem, such matrices are sums of permutation matrices. In a manner similar to the construction of standard Young tableaux, we introduce a path model for the construction of bipartite 1-factorizations and classify those of small order.

(14) **Dennis Eichhorn**, University of California Irvine
Email: deichhor@uci.edu
Title: Open problems involving cranks for partition congruences
Abstract: Dyson famously conjectured, correctly, that his rank statistic witnesses Ramanujan's first two congruences for $p(n)$, and that there exists a "crank" statistic that witnesses Ramanujan's congruence modulo 11 in a similar fashion. As it turns out, this phenomenon of congruence-witnessing statistics, which we now also call "cranks" in homage to Dyson, also occurs in other contexts within partition theory. In this talk, we give several open problems and conjectures in this area, highlighting some recent developments along the way.
This talk will include joint work with several coauthors.

(15) **Jinhui Fang**, Nanjing Normal University, China
Email: fangjinhui1114@163.com
Title: On bounded unique representation bases
Abstract: For a nonempty set A of integers and an integer n, let $r_A(n)$ be the number of representations of $n = a + a'$ with $a \leq a'$ and $a, a' \in A$, and let $d_A(n)$ be the number of representations of $n = a - a'$ with $a, a' \in A$. In 1941, Erdős and Turán posed the profound conjecture: If A is a set of positive integers such that $r_A(n) \geq 1$ for all sufficiently large n, then $r_A(n)$ is unbounded. In 2004, Nešetřil and Serra introduced the notion of bounded sets and confirmed the Erdős-Turán conjecture for all bounded bases. In 2003, Nathanson considered the existence of the set A with logarithmic

growth such that $r_A(n) = 1$ for all integers n. Recently, we prove that, for any positive function $l(x)$ with $l(x) \to 0$ as $x \to \infty$, there is a bounded set A of integers such that $r_A(n) = 1$ for all integers n and $d_A(n) = 1$ for all positive integers n, and $A(-x, x) \geq l(x) \log x$ for all sufficiently large x, where $A(-x, x)$ is the number of elements $a \in A$ with $-x \leq a \leq x$.
This is joint work with Prof. Yong-Gao Chen.

(16) **Daniel Benjamin Flores**, Purdue University
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Title:K-multimagic squares and magic squares of kth powers via the circle method
Abstract: Here we investigate *K-multimagic squares* of order N. These are $N \times N$ magic squares which remain magic after raising each element to the kth power for all $2 \leq k \leq K$. Given $K \geq 2$, we consider the problem of establishing the smallest integer $N_0(K)$ for which there exist *nontrivial* K-multimagic squares of order $N_0(K)$.

Previous results on multimagic squares show that $N_0(K) \leq (4K-2)^K$ for large K. We use the Hardy-Littlewood circle method to improve this to

$$N_0(K) \leq 2K(K+1) + 1.$$

The intricate structure of the coefficient matrix poses significant technical challenges for the circle method. We overcome these obstacles by generalizing the class of Diophantine systems amenable to the circle method and demonstrating that the multimagic square system belongs to this class for all $N \geq 4$. We additionally establish the existence of infinitely many $N \times N$ magic squares of distinct kth powers as soon as

$$N > 2 \min\{2^k, \lceil k(\log k + 4.20032) \rceil\}.$$

This result marks progress toward resolving an open problem popularized by Martin Gardner in 1996, which asks whether a 3×3 magic square of distinct squares exists.

(17) **Jonathan Fraser**, St. Andrews University, UK
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Title: Averages of the Fourier transform in finite fields
Abstract: Discrete Fourier analysis is a useful tool in various counting problems in vector spaces over finite fields. I will mention some results in this direction, with emphasis on a new approach based on quantifying Fourier decay via a spectrum of exponents coming from certain L^p averages.

(18) **Pedro A. Garcia Sánchez**, Universidad de Granada, Spain
Email: pedro@ugr.es
Title: Some problems related to the ideal class monoid of a numerical semigroup
Abstract: Let S be a numerical semigroup (a submonoid of the set of non-negative integers under addition such that $\max(\mathbb{Z} \setminus S)$ exists). A non-empty set of integers I is said to be an ideal of S if $I + S \subseteq I$ and I has a minimum. If I and J are ideals of S, we write $I \sim J$ if there exists an integer z such that $I = z + J$. The ideal class monoid of S is defined as the set of ideals of

S modulo this relation, where addition of two classes $[I]$ and $[J]$ is defined as $[I]+[J]=[I+J]$, with $I+J=\{i+j \mid i\in I, j\in J\}$.

An ideal I is said to be normalized if $\min(I)=0$. The set of normalized ideals of S, denoted by $\mathfrak{I}_0(S)$, is a monoid isomorphic to the ideal class monoid of S [1].

It is known that if S and T are numericals semigroups for which $\mathfrak{I}_0(S)$ is isomorphic to $\mathfrak{I}_0(T)$, then S and T must be the same numerical semigroup [4].

On $\mathfrak{I}_0(S)$ we can define a partial order $\preceq$ as $I \preceq J$ if there exists $K\in\mathfrak{I}_0(S)$ such that $I+K=J$. We know that if S and T are numerical semigroups with multiplicity three such that the poset $(\mathfrak{I}_0(S),\preceq)$ is isomorphic to the poset $(\mathfrak{I}_0(T),\preceq)$, then S and T are the same numerical semigroup [2]. However, if we remove the multiplicity three condition, this poset isomorphsm problem is still open.

In [3], we study the case when the poset $(\mathfrak{I}_0(S),\preceq)$ is a lattice. We show that this is the case if and only if the multiplicity of S is at most four.

References:
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2. P. A. Garcia Sánchez, The isomorphism problem for ideal class monoids of numerical semigroups, Semigroup Forum 108 (2024), 365–376.
3. S. Bonzio, P. A. Garcia Sánchez, The poset of normalized ideals of numerical semigroups with multiplicity three, to appear in Comm. Algebra.
4. S. Bonzio, P. A. Garcia Sánchez, When the poset of the ideal class monoid of a numerical semigroup is a lattice, arXiv:2412.07281.

(19) **Val Gladkova**, University of Cambridge, UK
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Title: A lower bound for the strong arithmetic regularity lemma
Abstract: The strong regularity lemma is a combinatorial tool originally introduced by Alon, Fischer, Krivelevich, and Szegedy in order to prove an induced removal lemma for graphs. Conlon and Fox showed that for some graphs, the strong regularity lemma must produce partitions of wowzer-size. This talk will sketch a proof that a comparable lower bound must hold for the arithmetic analogue of this lemma, in the setting of vector spaces over finite fields.

(20) **N. Hegyvári**, Eötvös University and Rényi Institute, Budapest
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Title: Variants of Raimi's theorem
Abstract: In 1968 Raimi proved the following intersection theorem:

There exists $E\subseteq\mathbb{N}$ such that, whenever $r\in\mathbb{N}$ and $\mathbb{N}=\bigcup_{i=1}^r D_i$ there exist $i\in\{1,2,\dots,r\}$ and $k\in\mathbb{N}$ such that $(D_i+k)\cap E$ is infinite and $(D_i+k)\setminus E$ is infinite.

A new proof of the theorem is due to N. Hindman, then to Bergelson and Weiss, and the generalization to the author. In the present talk, we give an outline of the new proofs and the generalization and some variations are discussed in different structures (e.g. in $\mathbf{Z}_n^k$, in $SL_2(\mathbb{F}_p)$.)

These variations are joint work with János Pach and Thang Pham.

(21) **Harald Helfgott**, CNRS/Institut de Mathématiques de Jussieu, France
Email: harald.helfgott@gmail.com
Title: Explicit estimates for sums of arithmetic functions, or the optimal use of finite information on Dirichlet series
Abstract: Let $F(s) = \sum_n a_n n^{-s}$ be a Dirichlet series. Say we have an analytic continuation of $F(s)$, and information on the poles of $F(s)$ with $|\Im s| \leq T$ for some large constant T. What is the best way to use this information to give explicit estimates on sums $\sum_{n \leq x} a_n$?

The problem of giving explicit bounds on the Mertens function $M(x) = \sum_{n \leq x} \mu(n)$ illustrates how open this basic question was. One might think that bounding $M(x)$ is essentially equivalent to estimating $\psi(x) = \sum_{n \leq x} \Lambda(n)$ or the number of primes $\leq x$. However, we have long had fairly satisfactory explicit bounds on $\psi(x) - x$, whereas bounding $M(x)$ well was a notoriously recalcitrant problem.

We give an optimal way to use information on the poles of $F(s)$ with $|\Im s| \leq T$. In particular, by means of our general, direct analytic approach, we give bounds on the Mertens function much stronger than those in the literature, while also substantially improving on estimates on $\psi(x)$ for x of moderate size.

We use functions of "Beurling-Selberg" type – namely, optimal approximant due to Carneiro-Littmann and an optional majorant/minorant due to Graham-Vaaler. Our procedure has points of contact with Wiener-Ikehara and also with work of Ramana and Ramar, but does not rely on results in the explicit analytic-number-theory literature.
Joint work with Andres Chirre.

(22) **Brian Hopkins**, Saint Peter's University
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Title: Scaled Arndt compositions
Abstract: In 2013, Jörg Ardnt observed that integer compositions $c_1 + c_2 + \cdots = n$ with $c_{2i-1} > c_{2i}$ for each positive i are counted by the Fibonacci numbers. This was confirmed by the speaker and Tangboonduangjit in 2022 and we explored generalizations of this pair-wise condition including $c_{2i-1} > c_{2i} + k$ for an affine parameter k. In the current work, a collaboration with Augustine Munagi, we consider scaling parameters, integers s and t, and resolve some cases of the general condition $sc_{2i-1} > tc_{2i} + k$. Techniques include generating functions and combinatorial proofs.

(23) **Robert Hough**, Stony Brook University
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Title: Lower order terms in the shape of cubic fields
Abstract: The ring of integers of a degree n number field may be viewed as an n-dimensional lattice within the canonical embedding. Spectrally expanding the space of lattices, we study the distribution of lattice shapes of rings of integers when cubic fields are ordered by discriminant by studying the Weyl sums testing the lattice shape against the real analytic Eisenstein

series and Maass cusp forms. In the case of Eisenstein series we identify a lower order main term of order $X^{11/12}$ when fields of discriminant of order X are counted with a smooth weight.
Joint work with Eun Hye Lee. Recent work of Lee and Ramin Tagloo-Bighash promises to extend these ideas to integral orbits in general prehomogeneous vector spaces.

(24) **Sophie Huczynska**, University of St. Andrews, UK
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Title: Additive triples in groups of odd prime order
Abstract: For a subset A of an additive group G, a Schur triple in A is a triple of the form $(a, b, a + b) \in A^3$. Denote by $r(A)$ the number of Schur triples of A; the behaviour of $r(A)$ as A ranges over subsets of a group G has been studied by various authors. When $r(A) = 0$, A is sum-free. The question of minimum and maximum $r(A)$ for A of fixed size in $\mathbb{Z}_p$ was resolved by Huczynska, Mullen and Yucas (2009) and independently by Samotij and Sudakov (2016). Several generalisations of the Schur triple problem have received attention. In this talk, I will present recent work (with Jonathan Jedwab and Laura Johnson) on the generalisation to triples $(a, b, a + b) \in A \times B \times B$, where $A, B \subseteq \mathbb{Z}_p$. Denote by $r(A, B, B)$ the number of triples of this form; we obtain a precise description of its full spectrum of values and show constructively that each value in this spectrum can be realised when B is an interval of consecutive elements in $\mathbb{Z}_p$.

(25) **Alex Iosevich**, University of Rochester
Email: iosevich@gmail.com
Title: The Fourier uncertainty principle, signal recovery, and applications
Abstract: We are going to discuss the analytic, arithmetic, and practical aspects of exact signal recovery, with the emphasis on the role of restriction theory for the Fourier transform and connections with the classical results of Bourgain, Talagrand, and others.

(26) **Robert Jacobs**
Email: bobbyjacobs@email.com
Title: Crossword puzzles
Abstract: It is known that the most words possible in a 15×15 crossword puzzle is 96 if the grid is symmetrical and connected and every word has at least 3 letters. In this talk, I will prove this and find the most words possible in other grids.

(27) **Thomas Karam**, University of Oxford, UK
Email: Thomas.Karam@maths.ox.ac.uk
Title: After the cap-set problem, and some properties of the slice rank
Abstract: The infamous cap-set problem asks for the size of the largest subset $A \subset \mathbb{F}_3^n$ not containing any solutions to the equation $x + y + z = 0$ aside from the trivial solutions $x = y = z$. A proof that that size is bounded above by C^n for some $C < 3$, which arose in 2016 in two breakthrough papers by Croot-Lev-Pach and by Ellenberg and Gijswijt (both published

in the Annals of Mathematics), was later reformulated by Tao in a more symmetric way, leading to the definition of a new notion of rank on tensors called the slice rank.

Since then, the slice rank has been studied further, and the resulting properties have often found related number-theoretic applications. To take the earliest and perhaps simplest example, a key component of the argument in the proof of the original cap-set problem itself is that the slice rank of a diagonal tensor is equal to its number of non-zero entries, mirroring the analogous property of matrix rank.

After reviewing some more such applications by other mathematicians, we will present some results concerning other basic properties of the slice rank, and in particular the ideas behind some of their simpler proofs in the special case where the support of the tensor is contained in an antichain: there, as established by Sawin and Tao, the slice rank of the tensor is equal to the smallest number of slices that suffice to cover its support. If time allows then we will also discuss how the proofs in this special case illuminate to some extent the proofs in the general case.

(28) **William Keith**, Michigan Technical University
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Title: $s \pmod{t}$-cores
Abstract: We consider simultaneous $(s, s+t, s+2t, \dots, s+pt)$-cores in the large-$p$ limit, or (when $s < t$), partitions in which no hook may be of length $s \pmod{t}$. As a boundary case of the general study made by Cho, Huh and Sohn, we find special symmetries and relations, such as generating functions, congruences when s is not coprime to t, and enumerations when s is coprime to t. Of particular interest is the comparison to the behavior of simultaneous (s, t)-cores and self-conjugate (s, t)-cores.
Joint work with Rishi Nath and James Sellers.

(29) **Gergo Kiss**, Budapest Corvinus University and Rényi Institute, Hungary
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Title: Weak tiling and the Coven-Meyerowitz conjecture from an analytic point of view
Abstract: The concept of weak tiling was originally introduced in $\mathbb{R}^n$ by Lev and Matolcsi, and has proven to be an essential tool in addressing Fuglede's conjecture for convex domains. In this talk, we extend the notion of weak tiling to the setting of cyclic groups and further generalize it using a natural averaging process. As a result, the tiles are no longer sets, but rather become step functions–a framework we refer to as functional tiling.

One advantage of this approach is that the cyclotomic divisors of the functions involved in a functional tiling remain the same as those of the characteristic functions of the original sets. Another is that functional tilings can be studied using the well-established tools and objective functions of linear programming, which is computationally efficient due to its polynomial-time solvability.

I will introduce the key quantities involved and present basic connections between functional and classical tilings. Finally, I will provide a counterexample to the Coven-Meyerowitz conjecture within the context of functional tilings. It is important to note, however, that none of the counterexamples we constructed in this setting correspond to tiling pairs of sets. Thus, the Coven-Meyerowitz conjecture for tiling sets remains open.
This is joint work with Itay Londner, Máté Matolcsi, and Gábor Somlai.

(30) **Sandor Kiss**, Budapest University of Technology and Economics, Hungary
Email: ksandor@math.bme.hu
Title: Generalized Stanley sequences
Abstract: For an integer $k \geq 3$, let $A_0 = \{a_1, \dots, a_t\}$ be a set of nonnegative integers which does not contain an arithmetic progression of length k. The set $S(A)$ is defined by the following greedy algorithm. If $s \geq t$ and $a_1, \dots, a_s$ have already been defined, then a_{s+1} is the smallest integer $a > a_s$ such that $\{a_1, \dots, a_s\} \cup \{a\}$ also does not contain a k-term arithmetic progression. The sequence $S(A)$ is called a *Stanley sequence* of order k generated by A_0. Starting out from a set of the form $A_0 = \{0, t\}$, Richard P. Stanley and Odlyzko tried to generate arithmetic progression-free sets by using the greedy algorithm. In 1999, Erdős, Lev, Rauzy, Sándor and Sárközy extended the notion of Stanley sequence to other initial sets A_0. In my talk I investigate some further generalizations of Stanley sequences and I give some density type results about them.
This is a joint work with Csaba Sándor and Quan-Hui Yang.

(31) **Yoshiharu Kohayakawa**, University of Sao Paulo, Brazil
Email: yoshi@ime.usp.br
Title: Arithmetic progressions in subset sums of sparse random sets of integers
Abstract: Given a set $S \subset \mathbb{N}$, its sumset $S + S$ is the set of all sums $s + s'$ with both s and s' elements of S. Given $p\colon \mathbb{N} \to [0, 1]$, let $A_n = [n]_p$ be the p-random subset of $[n] = \{1, \dots, n\}$: the random set obtained by including each element of $[n]$ in A_n independently with probability $p(n)$. Let $\varepsilon > 0$ be fixed, and suppose $p(n) \geq n^{-1/2+\varepsilon}$ for all large enough n. We prove that, then, with high probability, long arithmetic progressions exist in the sumset of any positive density subset of A_n, that is, with probability approaching 1 as $n \to \infty$, for any subset S of A_n with a fixed proportion of the elements of A_n, the sumset $S + S$ contains arithmetic progressions with $2^{\Omega(\sqrt{\log n})}$ elements. Joint work with Marcelo Campos and Gabriel Dahia.

(32) **Jakub Konieczny**, University of Oxford
Email: jakub.konieczny@gmail.com
Title: Multiplicative generalised polynomial sequences
Abstract: Generalised polynomials are sequences constructed from polynomial sequences using the integer part function, addition, and multiplication. Determining whether a given sequence is a generalised polynomial is often a non-trivial task. In joint work with J. Byszewski and B. Adamczewski,

we have discovered both surprising examples of such sequences and developed criteria to disprove that a given sequence is a generalised polynomial. More broadly, given a family of sequences, one can pose a classification problem: Which sequences in the family are generalized polynomials? In this talk, I will present a complete resolution of this problem for the family of multiplicative sequences, as well as partial results for (non-completely) multiplicative sequences.

(33) **Vjekoslav Kovač**, University of Zagreb, Croatia
Email: vjekovac@math.hr
Title: Several irrationality problems for Ahmes series
Abstract: Proving (ir)rationality of infinite series of distinct unit fractions has been an active topic of research for decades, with numerous occasional breakthroughs. We will investigate what can be obtained using elementary techniques (such as iterative constructions and the probabilistic method) and address several problems posed by Paul Erdős throughout the 1980s. In particular, we will study one type of irrationality sequences introduced by Erdős and Graham, (almost entirely) resolve a question by Erdős on simultaneous rationality of two or more "consecutive" series, and give a negative answer to an "infinite-dimensional" conjecture by Stolarsky.
This is joint work with Terence Tao (UCLA).

(34) **Emmanuel Kowalski**, ETH Zürich, Switzerland
Email: kowalski@ethz.ch
Title: Some pseudorandom graphs
Abstract: A classical construction associates to any Sidon set a graph without 4-cycles. We investigate some properties of these graphs in the case of the Sidon sets constructed by Forey, Fresán and myself using methods of algebraic geometry. In particular, this provides deterministic families of Ramanujan graphs with semi-circle and other interesting explicit asymptotic eigenvalue distributions.

Based on joint work with A. Forey and J. Fresán and discussions with Y. Wigderson and T. Schramm.

(35) **Noah Lebowitz-Lockard**
Email: nlebowi@gmail.com
Title: Partitions and ordered products
Abstract: Let $g(n)$ be the number of ways to express n as an ordered partition of numbers greater than 1. We also let $a(n)$ be the number of partitions of n of the form $n_1+n_2+\cdots+n_k$, where n_i is a multiple of n_{i+1} and the n_i are distinct. Though there is substantial research around $g(n)$, much less is known about $a(n)$. We discuss these two functions, as well as some new asymptotics on $a(n)$.

(36) **Paolo Leonetti**, Università degli Studi dell'Insubria, Italy
Email: leonettipaolo@gmail.com
Title: On the completeness induced by densities on natural numbers
Abstract: Let $\nu : \mathcal{P}(\mathbb{N}) \to \mathbb{R}$ be an "upper density" on the natural numbers

$\mathbb{N}$ (for instance, ν can be the upper asymptotic density or the upper Banach density). Then a natural pseudometric d_ν is induced on $\mathcal{P}(\mathbb{N})$, namely,

$$\forall A, B \subseteq \mathbb{N}, \quad d_\nu(A, B) := \nu(A \triangle B)$$

We provide necessary and sufficient conditions for the completeness of d_ν. Then we identify in which cases the latter ones are verified.

(37) **Mehdi Makhul**, London School of Economcs, UK
Email: mmakhul@risc.jku.at
Title: Web geometry and the orchard problem
Abstract: Let P be a set of n points in the plane, not all lying on a single line. The orchard planting problem asks for the maximum number of lines passing through exactly three points of P. Green and Tao showed that the maximum possible number of such lines for an n-element set is $\lfloor \frac{n(n-3)}{6} \rfloor + 1$. Lin and Swanepoel also investigated a generalization of the orchard problem in higher dimensions. Specifically, if P is a set of n points in d-dimensional space, they established an upper bound for the maximum number of hyperplanes passing through exactly $d+1$ points of P. Our goal is to describe the structural properties of configurations that achieve near-optimality in the asymptotic regime. Let $C \subset \mathbb{R}^d$ be an algebraic curve of degree r, and suppose that $P \subset C$ is a set of n points. If P determines at least cn^d hyperplanes, each passing through exactly $d+1$ points of P, then the following must hold: The degree of C must be $d+1$; and the curve C is the complete intersection of $\binom{d}{2} - 1$ quadric hypersurfaces. Our approach relies on the theory of web geometry and the Elekes-Szabó Theorem-a cornerstone of incidence geometry-both of which provide the structural basis for our analysis.
Joint work with Konrad Swanepoel.

(38) **Debyani Manna**, Indian Institute of Technology Roorkee, India
Email: d_manna@ma.iitr.ac.in, debyani97@gmail.com
Title: Some results on the extended inverse problem of $A + 2 \cdot A$
Abstract: Let A be a finite set of integers and $A+2\cdot A = \{a+2a' : a, a' \in A\}$. An extended inverse problem associated with the sumset $A + 2 \cdot A$ is to determine the underlying set A when the size of the sumset $A + 2 \cdot A$ deviates from the minimum possible size. We find all possible arithmetic structures of A for certain cardinalities of $A + 2 \cdot A$ and use them to address extended inverse problems in the Baumslag-Solitar group $BS(1, 2)$.
This is joint work with Ram Krishna Pandey.

(39) **Nathan McNew**, Towson University
Email: nmcnew@towson.edu
Title: The density of covering numbers
Abstract: In 1950, Erdős introduced covering systems–finite collections of arithmetic progressions whose union contains every integer. They featured in some of his favorite problems, many of which are still open. In 1979, answering one of Erdős's questions, Haight introduced covering numbers: positive integers n for which a covering system can be constructed with

distinct moduli that are divisors of n. If no proper divisor of n is a covering number, we call n a primitive covering number. We establish an upper bound on the number of primitive covering numbers, from which it follows that the set of covering numbers has a natural density. By refining techniques used to bound the density of abundant numbers, we obtain relatively tight bounds for the density of covering numbers and, in the process, improve the bounds on the density of abundant numbers as well.

(40) **Steve Miller**, Williams College
Email: sjm1@williams.edu
Title: Phase transitions for binomial sets under linear forms
Abstract: We generalize results on sum and difference sets of a subset S of $\mathbb{N}$ drawn from a binomial model. Given $A \subseteq \{0, 1, \ldots, N\}$, an integer $h \geq 2$, and a linear form $L : \mathbb{Z}^h \to \mathbb{Z}$

$$L(x_1, \ldots, x_h) \ := \ u_1 x_1 + \cdots + u_h x_h, \quad u_i \in \mathbb{Z}_{\neq 0} \text{ for all } i \in [h],$$

we study the size of

$$L(A) \ = \ \{u_1 a_1 + \cdots + u_h a_h : a_i \in A\}$$

and its complement $L(A)^c$ when each element of $\{0, 1, \ldots, N\}$ is independently included in A with probability $p(N)$, identifying two phase transitions. The first global one concerns the relative sizes of $L(A)$ and $L(A)^c$, with $p(N) = N^{-\frac{h-1}{h}}$ as the threshold. Asymptotically almost surely, below the threshold almost all sums generated in $L(A)$ are distinct and almost all possible sums are in $L(A)^c$, and above the threshold almost all possible sums are in $L(A)$. Our asymptotic formulae substantially extends work of Hegarty and Miller, resolving their conjecture. The second local phase transition concerns the asymptotic behavior of the number of distinct realizations in $L(A)$ of a given value, with $p(N) = N^{-\frac{h-2}{h-1}}$ as the threshold and identifies (in a sharp sense) when the number of such realizations obeys a Poisson limit. Our main tools are recent results on the asymptotic enumeration of partitions, Stein's method for Poisson approximation, and the martingale machinery of Kim-Vu.

(41) **Mohan**, BK Birla Institute of Engineering and Technology, India
Email: mohan98math@gmail.com
Title: Special additive complements of a set of natural numbers
Abstract: Let A be a set of natural numbers. A set B of natural numbers is said to be an additive complement of the set A if all sufficiently large natural numbers can be represented as $x + y$ for some $x \in A$ and $y \in B$. We shall describes various types of additive complements of the set A such as those additive complements of A that do or do not intersect A, additive complements which are the union of disjoint infinite arithmetic progressions, and additive complements having various densities etc. We estabilish that if $A = \{a_i : i \in \mathbb{N}\}$ is a set of natural numbers such that $a_i < a_{i+1}$ for $i \in \mathbb{N}$ and $\liminf_{n\to\infty}(a_{n+1}/a_n) > 1$, then there exists a set $B \subset \mathbb{N}$ such that $B \cap A = \varnothing$ and B is a sparse additive complement of the set A. Besides this, for a given positive real number $\alpha \leq 1$ and a finite set A, we investigate a set B such that B can be written as a union of disjoint

infinite arithmetic progressions with the natural density of $A+B$ equal to α.

(42) **Jeff Mozzochi**
Email: cjm@ix.netcom.com
Title: The closest known attempted proof of the twin prime conjecture
Abstract: Using a primitive formulation of the circle method we present a sufficient condition for the twin prime conjecture that misses being true by just an epsilon. We also show that the well-known sufficient condition for the twin prime conjecture implies the patently false statement that for each positive integer m, there exists an infinite number of prime pairs whose difference is m.

(43) **Rishi Nath**, York College (CUNY)
Email: rnath@york.cuny.edu
Title: Simultaneous (co)core partitions
Abstract: In the 1950s, Littlewood and others famously showed how to decompose an integer partition into its p-core and p-quotient for positive t. In the early 2000s, J. Anderson began the study of partitions which have both empty s-quotient and empty t-quotient for s and t relatively prime. Here we consider a perpendicular question, that of partitions which have both empty s-core and t-core.
This is joint work with T. Queer and A. Perez.

(44) **Mel Nathanson**, Lehman College (CUNY)
Email: melvyn.nathanson@lehman.cuny.edu
Title: Sizes of sumsets of finite sets of integers
Abstract: In the study of sums of finite sets of integers, most attention has been paid to sets with small sumsets (Freiman's theorem and related work) and to sets with large sumsets (Sidon sets and B_h-sets). The focus of this talk is on the full range of sizes of h-fold sums of a set of k integers. New results and open problems will be presented.

(45) **Vladyslav Oles**, University of Idaho
Email: voles@uidaho.edu
Title: Vanishing symmetric functions
Abstract: We continue an old solution by Noga Alon of conjectures of Arie Bialostocki. The first problem deals with vanishing symmetric functions on consecutive blocks in an arbitrary $\mathbb{Z}_n$-coloring of the positive integers. The second problem (unpublished) deals with vanishing symmetric functions on grid points inside of a polygon. The problems originated from the classical Theorem of Van der Waerden.
This is joint work with Arie Bialostocki.

(46) **Firdavs Rakhmonov**, University of St. Andrews, UK
Email: fr52@st-andrews.ac.uk
Title: Exceptional projections in finite fields: Fourier analytic bounds and incidence geometry
Abstract: We consider the problem of bounding the number of exceptional

projections (projections which are smaller than typical) of a subset of a vector space over a finite field. We establish bounds that depend on L^p estimates for the Fourier transform, improving various known bounds for sets with sufficiently good Fourier analytic properties. The special case $p = 2$ recovers a recent result of Bright and Gan (following Chen), which established the finite field analogue of Peres–Schlag's bounds from the continuous setting.
We prove several auxiliary results of independent interest, including a character sum identity for subspaces (solving a problem of Chen), and an analogue of Plancherel's theorem for subspaces. These auxiliary results also have applications in affine incidence geometry, that is, the problem of estimating the number of incidences between a set of points and a set of affine k-planes. We present a novel and direct proof of a well-known result in this area that avoids the use of spectral graph theory, and we provide simple examples demonstrating that these estimates are sharp up to constants.
This is joint work with Jonathan Fraser.

(47) **Asher Roberts**, St. Joseph's University New York
Email: aroberts@sjny.edu
Title: Large deviations of Selberg's central limit theorem on RH
Abstract: Assuming the Riemann hypothesis, we show that for $k > 0$ and $V \sim k \log\log T$,

$$\frac{1}{T}\,\text{meas}\left\{t \in [T, 2T] : \log|\zeta(1/2 + \text{i}t)| > V\right\} \leq C_k \frac{e^{-V^2/\log\log T}}{\sqrt{\log\log T}}.$$

This shows that Selberg's central limit theorem persists in the large deviation regime. As a corollary, we recover the result of Soundararajan and of Harper on the moments of ζ. This directly implies the sharp moment bounds of Soundararajan and Harper, i.e.,

$$\frac{1}{T}\int_T^{2T} |\zeta(1/2 + \text{i}t)|\text{d}t \leq C_k(\log T)^{k^2}.$$

This is joint work with Louis-Pierre Arguin (Oxford University) and Emma Bailey (University of Bristol).

(48) **David Ross**, University of Hawaii
Email: ross@math.hawaii.edu
Title: Upper density and a theorem of Banach
Abstract: Suppose A_n $(n \in \mathbb{N})$ is a sequence of sets in a finitely-additive measure space which are uniformly bounded away from 0, $\mu A_n \geq a > 0$ for all n. Then there is a subsequence A_{n_k}, where $\{n_k\}_k$ has upper Banach density $\geq a$, such that $\mu \bigcap_{k<N} A_{n_k} \geq a$ for every N. Surprisingly, this implies a density-limit version of a representation theorem of Banach:

Theorem: Let $\{f_n : n \in \mathbb{N}\}$ be a uniformly bounded sequence of functions on a set X. The following are equivalent: (i) $\{f_n\}_n$ weakly d-converges to 0; (ii) for any sequence $\{x_k : k \in \mathbb{N}\}$ in X, $d\text{-}\lim_{n\to\infty} \liminf_{k\to\infty} f_n(x_k) = 0$.

Here "d-" denotes a density limit. Banach's non-density version of this theorem (without the "d-") has been described by some as "marvelous".

(49) **Anne de Roton**, Université de Lorraine, Institut Elie Cartan, France
Email: anne.de-roton@univ-lorraine.fr
Title: Iterated sums races
Abstract: This is joint work with Paul Péringuey.
Our work provides a solution to a question posed by M. Nathanson in late 2024, but we later realized that this problem, along with an even more challenging one, had already been solved by N. Kravitz in a paper posted on arXiv in January 2025. While our construction is similar to his, it is simpler, and we hope that it can serve as an introductory step toward understanding the underlying ideas.
Nathanson's question is as follows:
For every integer $m \geq 3$, do there exist finite sets A and B of integers and an increasing sequence of positive integers $h_1 < h_2 < \cdots < h_m$, such that:

$$|h_i A| > |h_i B| \quad \text{if } i \text{ is odd,}$$
$$|h_i A| < |h_i B| \quad \text{if } i \text{ is even.}$$

Additionally, do there exist such sets with $|A| = |B|$? Can such sets be constructed with $|A| = |B|$ and $diam(A) = diam(B)$?
We provide a positive answer to these questions and propose an iterative construction of sets that satisfy these conditions.

(50) **Akash Singha Roy**, University of Georgia
Email: akash01s.roy@gmail.com
Title: Joint distribution in residue classes of families of multiplicative functions
Abstract: The distribution of values of arithmetic functions in residue classes has been a problem of great interest in elementary, analytic, and combinatorial number theory. In work studying this problem for large classes of multiplicative functions, Narkiewicz obtained general criteria deciding when a family of such functions is jointly uniformly distributed among the coprime residue classes to a fixed modulus. Using these criteria, he along with Śliwa, Rayner, Dobrowolski, Fomenko, and others, gave explicit results on the distribution of interesting multiplicative functions and their families in coprime residue classes.

In this talk, we shall give best possible extensions of Narkiewicz's criteria (and hence also of the other aforementioned results) to moduli that are allowed to vary in a wide range. This is motivated by the celebrated Siegel-Walfisz theorem on the distribution of primes in arithmetic progressions, and our results happen to be some of the best possible qualitative analogues of the Siegel-Walfisz theorem for the classes of multiplicative functions considered by Narkiewicz and others. Our arguments blend ideas from multiple subfields of number theory, as well as from linear algebra over rings, commutative algebra, and arithmetic and algebraic geometry. This talk is partly based on joint work with Paul Pollack.

(51) **Cihan Sabuncu**, Université de Montréal, Canada
Email: cihan.sabuncu@umontreal.ca

Title: Extreme values of $r_3(n)$ in arithmetic progressions
Abstract: A classical result of Chowla shows that the representation function $r_3(n)$, which counts the number of ways n can be expressed as a sum of three squares, satisfies

$$r_3(n) \gg \sqrt{n} \log\log n$$

for infinitely many integers n. This lower bound, in turn, also implies that $L(1, \chi_D) \gg \log\log |D|$ holds for infinitely many fundamental discriminants $D < 0$. In this talk, we will investigate whether such extremal behavior of $r_3(n)$ persists when n is restricted to lie in an arithmetic progression $n \equiv a \pmod{q}$.
This is joint work with Jonah Klein and Michael Filaseta.

(52) **Vincent Schinina**, CUNY Graduate Center
Email: vschinina@gradcenter.cuny.edu
Title: On a missing interval of integers from $\mathcal{R}_{\mathbf{Z}}(h, 4)$
Abstract: The set $\mathcal{R}_{\mathbf{Z}}(h, 4)$ consists of all possible sizes for the h-fold sumset of sets containing four integers. An immediate question to ask is what are the elements of this set? We know that $\mathcal{R}_{\mathbf{Z}}(h, 4) \subseteq [3h+1, \binom{h+3}{h}]$, where the right side is an interval of integers that includes the endpoints. These endpoints are known to be attained. By observation, it appears that the interval of integers $[3h + 2, 4h - 1]$ is absent from $\mathcal{R}_{\mathbf{Z}}(h, 4)$. We will briefly discuss the procedure used to prove that the integers in $[3h + 2, 4h - 1]$ are not possible sizes for the h-fold sumset of a set containing four integers. Furthermore, we will confirm that this interval can't be made larger by exhibiting a set whose h-fold sumset has size $4h$.

(53) **Alisa Sedunova**, Purdue University
Email: alisa.sedunova@gmail.com
Title: The multiplication table constant and sums of two squares
Abstract: Let $r_1(n)$ be the number of representations of n as the sum of a square and a square of a prime. We discuss the erratic behavior of r_1, which is similar to the one of the divisor function. We will show that the number of integers up to x that have at least one such representation is asymptotic to $(\pi/2)x \log x$ minus a secondary term of size $x/(\log x)^{1+d+o(1)}$, where d is the multiplication table constant. Detailed heuristics suggest very precise asymptotic for the secondary term as well. In particular, our proofs imply that the main contribution to the mean value of $r_1(n)$ comes from integers with unusual number of prime factors, i.e. those with $\omega(n) \sim 2 \log\log x$ (for which $r_1(n) \sim (\log x)^{\log 4-1}$), where $\omega(n)$ is the number of district prime factors of n.
In the talk we will review the results of several works that include a recent joint preprint with Andrew Granville and Cihan Sabuncu and my paper from 2022 as well as some work in progress.

(54) **James A. Sellers**, University of Minnesota Duluth
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Title: Extending congruences for overpartitions with ℓ-regular non-overlined parts

Abstract: Recently, Alanazi, Alenazi, Keith, and Munagi considered overpartitions wherein the non-overlined parts must be ℓ-regular, that is, the non-overlined parts cannot be divisible by the integer ℓ. In the process, they proved a general parity result for the corresponding enumerating functions, and they also proved some specific Ramanujan–like congruences for the case $\ell = 3$. In this talk, we use elementary generating function manipulations to significantly extend the set of known congruences for these functions.

All of the proof techniques used herein are elementary, relying on classical q-series identities and generating function manipulations, along with mathematical induction.

(55) **Steven Senger**, Missouri State University
Email: StevenSenger@missouristate.edu
Title: VC-dimension of subsets of the Hamming graph
Abstract: Vapnik-Chervonenkis or VC-dimension has been a useful tool in combinatorics, machine learning, and other areas. Given a graph from a well-studied family, there has been recent activity on size thresholds for a subset of a graph to guarantee bounds on the VC-dimension of the subset. These resemble finite point configuration results, such as the Erdos-Falconer distance problem, both in form as well as in the techniques of proof. Typically, one looks at graphs that are highly pseudorandom, such as the distance graph or the dot product graph, but the Hamming graph is quantifiably less pseudorandom, and standard techniques seem to break down and yield very weak results if any. We present a suite of results that outperform their counterparts for the Hamming graph. The proofs are completely elementary, and in some cases, tight.

(56) **Besfort Shala**, University of Bristol, UK
Email: besfort.shala@bristol.ac.uk
Title: Multiplicative energy in number theory
Abstract: I will discuss the important role of multiplicative energy of sets in number theory, particularly in the probabilistic theory of random multiplicative functions. The aim is to provide a survey of recent results in the area.

(57) **Ilya Shkredov**, Purdue University
Email: ilya.shkredov@gmail.com
Title: Some applications of the higher energy method to distribution irregularities
Abstract: We review recent results obtained by the method of higher sumsets and higher energies. In particular, we discuss two applications: irregularities in the distribution of the difference set and irregularities in the large Fourier coefficients of sets with small sumsets.

(58) **Gábor Somlai**, Eötvös Loránd University and Rényi Institute, Hungary
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Title: Pushing the gap between tiles and spectral sets even further
Abstract: Fuglede conjectured that a bounded measurable set in a locally

compact topological space endowed with Haar measure is spectral if and only if it is a tile and Fuglede also confirmed the conjecture for sets whose tiling complement is a lattice and for spectral sets one of whose spectrums is a lattice.

The conjecture was disproved by Tao in the case of finite abelian groups where the counting measure plays the role of the Haar measure. Tao constructed a spectral set in $\mathbb{Z}_3^5$ of size 6, that is not a tile. This construction was lifted to the 5 dimensional Euclidean space, where the original conjecture was mostly studied.

Lev and Matolcsi verified Fuglede's conjecture for convex sets in $\mathbb{R}^n$ for every positive integer n. The key of proving the harder direction of the conjecture is to introduce the weak tiling property and prove that all spectral sets are weak tilings.

One of the goals of our work was to answer a question of Kolountzakis, Lev and Matolcsi, whether there is a weak tile that is neither a tile nor spectral. There is such a set which apparently makes it harder to prove the spectral-tile direction of the conjecture in the remaining open cases.

The other result towards structurally distinguishing spectral sets and tiles was a disproof of a conjecture of Greenfeld and Lev. They conjectured that the product of two sets is spectral if and only if both of them are spectral. A similar property holds for tiles, but the product of a non-spectral set with a spectral set can be spectral. Finally, we obtain an easy characterization of tiles using the spectral property.

(59) **Christoph Spiegel**, Zuse Institute Berlin, Germany
Email: spiegel@zib.de
Title: An unsure talk on an un-Schur problem
Abstract: Graham, Rödl, and Ruciński originally posed the problem of determining the minimum number of monochromatic Schur triples that must appear in any 2-coloring of the first n integers. This question was subsequently resolved independently by Datskovsky, Schoen, and Robertson and Zeilberger. Here we suggest studying a natural anti-Ramsey variant of this question and establish the first non-trivial bounds by proving that the maximum fraction of Schur triples that can be rainbow in a given 3-coloring of the first n integers is at least 0.4 and at most 0.66656. We conjecture the lower bound to be tight. This question is also motivated by a famous analogous problem in graph theory due to Erdős and Sós regarding the maximum number of rainbow triangles in any 3-coloring of K_n, which was settled by Balogh, et al.
This is joint work with Olaf Parczyk.

(60) **Josiah Sugarman**, Hebrew University of Jerusalem, Israel
Email: josiahsugarman@gmail.com
Title: Explicit spectral gap for the quaquaversal operator
Abstract: The spectral gap of an operator is the gap between the largest eigenvalue and the rest of the spectrum. In the mid 90s, John Conway and Charles Radin introduced a three dimensional substitution tiling, the Quaquaversal Tiling, with the property that the orientations of its tiles

equidistribute faster than what is possible for two dimensional substitution tilings. Conway and Radin showed that the orientations of the tiles were dense in $SO(3)$ and implicity introduced an operator (later explicitly studied by Draco, Sadun, and Van Wieren) whose spectral gap controls the equidistribution rate. Draco, Sadun, and Van Wieren studied the eigenvalues of this operator numerically and conjectured that it has a spectral gap bounded below by approximately 0.0061697. We exploit a fact, due to Serre, that the group of orientations for this tiling is 2-arithmetic and follow a strategy similar to Lubotzky, Phillips, and Sarnak's in order to obtain a lower bound of about 0.0061711, resolving the conjecture.

(61) **Christian Táfula**, Université de Montréal, Canada
Email: christian.tafula.santos@umontreal.ca
Title: Waring and Waring–Goldbach subbases with prescribed representation functions
Abstract: We investigate representation functions $r_{A,h}(n)$ of subsets A of k-th powers $\mathbb{N}^k$ and k-th powers of primes $\mathbb{P}^k$. Building on work of Vu, Wooley, and others, we prove that for $h \geq h_k = O(8^k k^2)$ and regularly varying $F(n)$ satisfying $\lim_{n\to\infty} F(n)/\log n = \infty$, there exists $A \subseteq \mathbb{N}^k$ such that

$$r_{A,h}(n) \sim \mathfrak{S}_{k,h}(n)F(n),$$

where $\mathfrak{S}_{k,h}(n)$ is the singular series associated to Waring's problem. In the case of prime powers, we obtain analogous results for $F(n) = n^\kappa$. For $F(n) = \log n$, we prove that for every $h \geq 2k^2(2\log k + \log\log k + O(1))$, there exists $A \subseteq \mathbb{P}^k$ such that $r_{A,h}(n) \asymp \log n$, showing the existence of thin subbases of prime powers.

(62) **Johann Thiel**, New York City College of Technology (CUNY)
Email: jthiel@citytech.cuny.edu
Title: Bivariate polynomials associated with binary trees created by QuickSort
Abstract: In this talk we describe a generating series whose coefficients are polynomials that, for a given positive integer n, encode the depth at which the various list entries appear as labeled nodes in the binary trees obtained by QuickSorting permutations of the list consisting of one copy of each of the first n non-negative integers. Extracting the appropriate coefficients yields information for the number of times a given list entry appears at a given depth, the total number of list entries that appear at a given depth, and consequently the average number of list entries that appear at a given depth taken over all $n!$ permutations. Joint work with David M. Bradley.

(63) **Salvatore Tringali**, Hebei Normal University, China
Email: salvo.tringali@gmail.com
Title: Power monoids and the Bienvenu-Geroldinger problem for torsion groups
Abstract: Let M be a (multiplicatively written) monoid with identity element 1_M. Endowed with the operation of setwise multiplication induced by M, the collection of finite subsets of M containing 1_M forms a monoid in

its own right, denoted by $\mathcal{P}_{\text{fin},1}(M)$ and called the reduced finitary power monoid of M.

It is natural to ask whether, for all H and K in a given class of monoids, $\mathcal{P}_{\text{fin},1}(H)$ is isomorphic to $\mathcal{P}_{\text{fin},1}(K)$ if and only if H is isomorphic to K. Originating from a conjecture of Bienvenu and Geroldinger recently settled by Yan and myself, the problem — together with its numerous variants and ramifications — has non-trivial connections to additive number theory and related fields. In this talk, I will present a positive solution for the class of torsion groups.

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(64) **Trevor D. Wooley**, Purdue University
Email: twooley@purdue.edu
Title: Equidistribution and L^p-sets for $p < 2$

Abstract: We investigate subsets $\mathcal{A}$ of the natural numbers having the property that, for some positive number $p < 2$, one has

$$\int_0^1 \Big| \sum_{n \in \mathcal{A} \cap [1,N]} e(n\alpha) \Big|^p \, \mathrm{d}\alpha \ll |\mathcal{A} \cap [1,N]|^p N^{\varepsilon - 1}.$$

Examples of such sets include (but are not restricted to) the squarefree, or more generally, the r-free numbers. For polynomials $\psi(x;\boldsymbol{\alpha}) = \alpha_k x^k + \ldots + \alpha_1 x$, having coefficients α_i satisfying suitable irrationality conditions, we show that the sequence $(\psi(n;\boldsymbol{\alpha}))_{n \in \mathcal{A}}$ is equidistributed modulo 1.

Additional abstracts

(1) **Adrian Beker** University of Zagreb, Croatia
Email: adrian.beker@math.hr
Title: The Erdős-Moser sum-free set problem via improved bounds for k-configurations
Abstract: In a recent remarkable breakthrough, Kelley and Meka significantly improved the bounds for sets of integers lacking three-term arithmetic progressions. Soon thereafter, Filmus, Hatami, Hosseini and Kelman extended their methods to sets lacking so-called binary systems of linear forms. Central to their approach is a new sparse graph counting lemma which is tailored to density increment arguments in additive number theory. In this talk, I will discuss how one can combine improvements in certain quantitative aspects of their graph counting lemma with further Kelley–Meka-style arguments to get new bounds for sets lacking k-configurations, i.e. collections of k integers together with their pairwise arithmetic means. As a consequence, this gives an alternative proof of a bound for the Erdős–Moser sum-free set problem of best known shape.

(2) **Krystian Gajdzica**, Jagiellonian University, Poland
Email: krystian.gajdzica@uj.edu.pl
Title: On the Bessenrodt-Ono inequality for polynomials
Abstract: In 2016, Bessenrodt and Ono discovered that the partition function satisfies the ineqaulity of the form

$$p(a)p(b) > p(a+b)$$

for all integers $a, b \geqslant 2$ with $a+b > 9$. That result initiated a broad research related to investigating analogue properties for other partition statistics. Moreover, it turns out that we can pass from the discrete problem to the continuous one by considering appropriate family of polynomials. That was the idea of Heim and Neuhauser, who showed that a family of polynomials defined via generating function:

$$\sum_{n=0}^{\infty} P_n(x)q^n := \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^x}$$

fulfills the Bessenrodt-Ono type inequality.

In this talk, we present an extension of the above and examine a family of polynomials associated to the so-called A-partition function in particular, and a sequence of real numbers in general.

(3) **Brad Isaacson**, New York City College of Technology (CUNY)
Email: bisaacson@citytech.cuny.edu
Title: Reciprocity formulae for generalized Dedekind and Hardy-Berndt sums involving Apostol-Bernoulli polynomials and functions
Abstract: This talk is concerned with generalized Dedekind and Hardy–Berndt sums and their corresponding reciprocity theorems. It emerges that the ingredients needed for certain reciprocity theorems are functions possessing suitable product formulas and Raabe-type multiplication formulas, for which the Bernoulli and Euler functions both enjoy. However, many

functions possess these properties, and consequently, generalized sums involving these functions must also obey a reciprocity theorem of this type. In this talk, we consider a unification of the generalized Dedekind and Hardy–Berndt sums which involve Apostol–Bernoulli functions and present its reciprocity theorem which contains all of the reciprocity theorems for the generalized Dedekind and Hardy-Berndt sums in the literature as special cases.

(4) **Bruce Keener**, University of Findlay
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Title: Third order recurrence relations with Sierpiński and Riesel numbers
Abstract: A Sierpiński number is an odd positive integer k satisfying $k \cdot 2^n + 1$ is composite for every positive integer n. In a similar fashion, a Riesel number is an odd positive integer k satisfying $k \cdot 2^n - 1$ is composite for every positive integer n. Previous work has been done to show an intersection between Sierpiński or Riesel numbers with various number sequences such as Fibonacci, Lucas numbers, images of various polynomials, polygonal numbers, Carmichael numbers, Ruth-Aaron pairs, binomial coefficients, and Narayanas cow. We expand on these findings by considering such an intersection with the Tribonacci, Padovan, Perrin, Van der Laan, and Leonardo sequences.